

OFFICIAL COURSE STUDY GUIDE

SAT & ACT Math Foundations

Rebuild the algebra, geometry, and number skills that every college admissions test assumes.

TEST PREP · BEGINNER · 8 MODULES · 9 HR 1 MIN

Course by The Organic Chemistry Tutor · Curated and published by MegaCourse

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SAT & ACT Math Foundations: The Complete Study Guide

2026 Edition

Published by MegaCourse · megacourse.app

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INTRODUCTION

About This Course

Welcome to the official study guide for "SAT & ACT Math Foundations." This course provides a complete rebuild of the algebra, geometry, and number skills essential for college admissions tests. It prepares you for the math sections of the SAT and ACT.

Many students approach these tests needing a refresher on fundamental math concepts. This course addresses that need directly. It covers the core mathematical principles that these exams consistently assess, ensuring you have a solid understanding.

Your instructor, The Organic Chemistry Tutor, presents these topics clearly and effectively. The course structure, with 8 modules and 48 lessons, guides you through each concept systematically. This guide supplements the video instruction, offering additional support for your learning.

Course Facts

FORMAT	8 modules · 48 lessons
VIDEO INSTRUCTION	About 9 hr 1 min
LEVEL	Beginner
CATEGORY	Test Prep
INSTRUCTOR	The Organic Chemistry Tutor
CERTIFICATE	Included with course completion on MegaCourse
COURSE HOME	megacourse.app/courses/sat-and-act-math-foundations

What You Will Learn

- Rebuild core algebra: linear equations, systems, exponents, and quadratics.
- Refresh the geometry and data-analysis facts admissions tests recycle.
- Develop calculator-smart and calculator-free solving habits.
- Prepare a foundation strong enough for any SAT or ACT strategy course.

Who This Course Is For

This course is designed for any student preparing for the SAT or ACT who needs to strengthen their mathematical foundations. It is ideal for beginners or those seeking a thorough review of core algebra, geometry, and data analysis skills.

No prior advanced mathematical knowledge is assumed. The course starts from the ground floor, building your understanding step by step. It ensures you are well-prepared before moving on to test-specific strategies.

Before You Start

- No prior experience is required.

How to Use This Guide

To get the most from this course and guide, combine your video lessons with active study. Watch each video lesson first to grasp the concepts. Then, use this study guide to reinforce your learning, work through examples, and practice problems.

Pace yourself through the 8 modules and 48 lessons. Consistent daily or weekly study is more effective than cramming. Utilize the module outlines to track your progress and review topics as needed.

This guide includes additional exercises and explanations. Use them to solidify your understanding. The goal is not just to watch the videos, but to actively engage with the material and apply what you learn.

CHAPTER 1

Algebra Foundations for SAT Math

This chapter rebuilds essential algebra skills necessary for success on college admissions tests like the SAT and ACT. We start with fundamental concepts, including how to evaluate algebraic expressions and functions. You will learn to solve various types of equations, from basic linear equations to more complex quadratic, rational, and radical forms. The chapter also covers techniques for solving systems of equations and identifying undefined values, providing a strong algebraic base for advanced problem-solving.

Mastering these foundational skills is critical for navigating the math sections of college admissions tests. Each topic is presented with clear explanations and step-by-step examples, enabling you to practice and solidify your understanding. By the end of this chapter, you will have a robust set of algebraic tools, preparing you for more intricate problems and strategies in subsequent modules.

Foundations of Algebraic Expressions and Equations

Covers: Evaluating Functions and Solving Basic Equations - Algebra - SAT Math Part 1 · SAT Math Part 5 - Evaluating Algebraic Expressions With Multiple Variables - Membership · Identifying Undefined Function Values - SAT Math Part 8

The foundation of algebra rests on understanding **expressions**, **equations**, and **functions**. An **algebraic expression** combines variables, numbers, and operation symbols. An **equation** sets two expressions equal to each other. A **function** is a special type of relationship where each input has exactly one output.

Evaluating Expressions and Functions

To evaluate an algebraic expression or function, substitute the given numerical values for the variables and then simplify the result using the order of operations (Parentheses, Exponents, Multiplication and Division from left to right, Addition and Subtraction from left to right).

Consider the expression $2x^2 - 3y$. If $x = 3$ and $y = -2$, substitute these values:

$$\begin{aligned} &2(3)^2 - 3(-2) \\ &= 2(9) - (-6) \\ &= 18 + 6 \\ &= 24 \end{aligned}$$

Function notation, such as $f(x)$, indicates that the output depends on the input x . To evaluate $f(x)$ for a specific value, replace x with that value. For example, if $f(x) = 2x - 5$, finding $f(4)$ means:

$$\begin{aligned} f(4) &= 2(4) - 5 \\ &= 8 - 5 \\ &= 3 \end{aligned}$$

Solving Basic Equations

Solving an equation means finding the value(s) of the variable(s) that make the equation true. For basic linear equations with one variable, the goal is to isolate the variable on one side of the equation. Perform inverse operations to both sides of the equation to maintain balance.

For example, to solve $3x + 7 = 19$:

1. Subtract 7 from both sides:

$$\begin{aligned}3x + 7 - 7 &= 19 - 7 \\3x &= 12\end{aligned}$$

2. Divide both sides by 3:

$$\begin{aligned}3x / 3 &= 12 / 3 \\x &= 4\end{aligned}$$

To check your solution, substitute the value back into the original equation: $3(4) + 7 = 12 + 7 = 19$. Since $19 = 19$, the solution is correct.

Identifying Undefined Function Values

Certain algebraic expressions become **undefined** for specific values of their variables. On tests, this often occurs in two main scenarios:

1. **Division by zero:** An expression with a variable in the denominator is undefined when the denominator equals zero. For example, the expression $1 / (x - 3)$ is undefined when $x - 3 = 0$, which means $x = 3$. To find undefined values, set the denominator equal to zero and solve for the variable.
2. **Even root of a negative number:** Expressions involving square roots (or any even root) are undefined in the real number system if the value under the radical sign is negative. For example, $\sqrt{x - 5}$ is undefined if $x - 5 < 0$, which means $x < 5$. The expression is defined only for $x \geq 5$. To find values that make an even root undefined, set the expression under the radical less than zero and solve.

Always be mindful of these restrictions when working with algebraic expressions and functions. Failing to identify undefined values can lead to incorrect solutions or misinterpretations of problem constraints. Practice identifying these critical points to avoid common pitfalls on exams.

Mastering Factoring and Quadratic Equations

Covers: Solving Quadratic Equations By Factoring - SAT Math Part 2 · Solving Rational Equations and Factoring Trinomials - SAT Math Part 3 · Factoring Perfect Square Trinomials - SAT Math Part 6

Quadratic equations are equations of the form $ax^2 + bx + c = 0$, where a , b , and c are constants and a is not zero. One of the most common methods for solving quadratic equations on college

admissions tests is **factoring**. Factoring involves rewriting an expression as a product of simpler expressions (factors). The **Zero Product Property** states that if the product of two or more factors is zero, then at least one of the factors must be zero.

Factoring Techniques

- Greatest Common Factor (GCF):** Always look for a GCF first. This is the largest term that divides into all terms of the expression. Example: $3x^2 + 6x = 3x(x + 2)$
- Factoring Trinomials ($x^2 + bx + c$):** For trinomials where the leading coefficient (a) is 1, find two numbers that multiply to c and add to b . Example: $x^2 + 5x + 6$ We need two numbers that multiply to 6 and add to 5. These are 2 and 3. So, $x^2 + 5x + 6 = (x + 2)(x + 3)$
- Factoring Trinomials ($ax^2 + bx + c$):** When a is not 1, use methods like grouping or "guess and check". Example: $2x^2 + 7x + 3$ We look for factors of $2x^2$ (like $2x$ and x) and factors of 3 (like 1 and 3) that, when combined, produce the middle term $7x$. $(2x + 1)(x + 3)$ Check: $2x \cdot x + 2x \cdot 3 + 1 \cdot x + 1 \cdot 3 = 2x^2 + 6x + x + 3 = 2x^2 + 7x + 3$. This is correct.
- Factoring Perfect Square Trinomials:** These are trinomials that result from squaring a binomial. They follow the patterns:
 - $a^2 + 2ab + b^2 = (a + b)^2$
 - $a^2 - 2ab + b^2 = (a - b)^2$ Example: $x^2 + 6x + 9$ Here, $a = x$ and $b = 3$. Since $2ab = 2(x)(3) = 6x$, this is a perfect square trinomial. So, $x^2 + 6x + 9 = (x + 3)^2$
- Difference of Squares:** This pattern is $a^2 - b^2 = (a - b)(a + b)$. Example: $x^2 - 16$ Here, $a = x$ and $b = 4$. So, $x^2 - 16 = (x - 4)(x + 4)$

Solving Quadratic Equations by Factoring

Once a quadratic equation is factored, apply the Zero Product Property. Set each factor equal to zero and solve for the variable.

Consider the equation $x^2 + 5x + 6 = 0$:

- Factor the quadratic expression:

$$(x + 2)(x + 3) = 0$$

- Set each factor equal to zero:

$$x + 2 = 0 \text{ OR } x + 3 = 0$$

- Solve each linear equation:

$$x = -2 \text{ OR } x = -3$$

The solutions (also called roots or zeros) are $x = -2$ and $x = -3$. Always ensure the quadratic equation is set to zero before factoring. If it is not, rearrange the terms first. Factoring is a powerful technique for

solving quadratics efficiently, especially when the factors are integers.

Solving Advanced Algebraic Equations and Systems

Covers: Solving Rational Equations and Factoring Trinomials - SAT Math Part 3 · Solving Radical Equations and Plugging In Numbers - SAT Math Part 4 · SAT Math Part 7 - Solving Systems of Equations Using Elimination & Substitution - Membership

This section extends equation-solving skills to more complex forms: rational equations, radical equations, and systems of linear equations. It also introduces a valuable test-taking strategy, "Plugging In Numbers."

Solving Rational Equations

Rational equations involve algebraic fractions, where variables may appear in the denominators. To solve them, eliminate the denominators by multiplying all terms by the **least common denominator (LCD)** of all fractions in the equation.

Consider $\frac{1}{x} + \frac{1}{2} = \frac{3}{(2x)}$:

1. The LCD is $2x$. Multiply every term by $2x$:

$$\begin{aligned} 2x * (\frac{1}{x}) + 2x * (\frac{1}{2}) &= 2x * (\frac{3}{(2x)}) \\ 2 + x &= 3 \end{aligned}$$

2. Solve for x :

$$x = 1$$

Always check for **extraneous solutions**. These are values that make the original equation undefined (usually by causing a denominator to be zero). For $x = 1$, no denominator is zero, so it is a valid solution. If $x = 0$ had been a solution, it would be extraneous.

Solving Radical Equations

Radical equations contain a variable under a radical sign, typically a square root. To solve these, isolate the radical term on one side of the equation. Then, raise both sides to the power corresponding to the index of the radical. For a square root, square both sides.

Consider $\sqrt{x + 2} = 3$:

1. The radical $\sqrt{x + 2}$ is isolated.
2. Square both sides:

$$\begin{aligned} (\sqrt{x + 2})^2 &= 3^2 \\ x + 2 &= 9 \end{aligned}$$

3. Solve for x :

$$x = 7$$

Check for **extraneous solutions** by substituting $x = 7$ back into the original equation: $\sqrt{7 + 2} = \sqrt{9} = 3$. Since $3 = 3$, the solution is valid.

Solving Systems of Equations

A **system of equations** consists of two or more equations with two or more variables. The goal is to find values for all variables that satisfy all equations simultaneously. Two primary methods are **substitution** and **elimination**.

1. Substitution Method:

- Solve one equation for one variable.
- Substitute that expression into the second equation.
- Solve the resulting single-variable equation, then find the other variable.

Example:

$$\begin{aligned}y &= x + 1 \\ 2x + y &= 7\end{aligned}$$

Substitute $(x + 1)$ for y in the second equation:

$$\begin{aligned}2x + (x + 1) &= 7 \\ 3x + 1 &= 7 \\ 3x &= 6 \\ x &= 2\end{aligned}$$

Substitute $x = 2$ into $y = x + 1$ to get $y = 3$. The solution is $(2, 3)$.

2. Elimination Method:

- Multiply one or both equations to make the coefficients of one variable opposites.
- Add the equations to eliminate that variable.
- Solve the resulting single-variable equation, then find the other variable.

Example:

$$\begin{aligned}x + y &= 5 \\ x - y &= 1\end{aligned}$$

Add the two equations:

$$\begin{aligned}(x + y) + (x - y) &= 5 + 1 \\ 2x &= 6 \\ x &= 3\end{aligned}$$

Substitute $x = 3$ into $x + y = 5$ to get $y = 2$. The solution is $(3, 2)$.

Plugging In Numbers (P.I.N.)

This is a powerful strategy for multiple-choice questions, especially with abstract algebraic expressions. Substitute simple numbers for variables in the problem and calculate the result. Then, substitute the same numbers into each answer choice to find the match.

Example: If $x = 3$, what is the value of $(x - 2)(x + 2)$? (A) $x^2 - 4$ (B) $x^2 + 4$ (C) x^2 (D) $2x$

1. Choose $x = 3$.
2. Evaluate the original expression: $(3 - 2)(3 + 2) = (1)(5) = 5$.
3. Evaluate answer choice (A): $3^2 - 4 = 9 - 4 = 5$. This matches. The correct answer is (A). This strategy can save time and reduce algebraic errors. Choose numbers carefully to avoid ambiguity.

KEY TAKEAWAYS

- Evaluating expressions and functions involves substituting values and following the order of operations.
- Basic equations are solved by isolating the variable using inverse operations on both sides.
- Expressions are undefined when a denominator is zero or an even root contains a negative number.
- Factoring quadratic expressions allows you to solve quadratic equations using the Zero Product Property.
- Rational and radical equations require specific steps to eliminate fractions or radicals, and solutions must be checked for extraneous values.
- Systems of linear equations can be solved efficiently using either the substitution or elimination method.
- The "Plugging In Numbers" strategy is a valuable tool for quickly solving certain multiple-choice problems by testing numerical examples.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. If $f(x) = x^2 - 3x + 1$, what is the value of $f(-2)$?
2. For what value of x is the expression $(x + 5) / (2x - 8)$ undefined?
3. Solve the quadratic equation $x^2 - 7x + 10 = 0$ by factoring.
4. Solve the radical equation $\sqrt{3x - 5} = 4$.
5. Use the elimination method to solve the system of equations: $2x + y = 10$ and $x - y = 2$.

CHAPTER 2

Algebra Foundations: Exponents, Functions, and Special Equations

Chapter 2 strengthens your algebraic foundation, essential for success on the SAT and ACT. This chapter reviews fundamental rules for exponents, then moves into solving absolute value equations and linear inequalities. You will learn to work with composite functions and manipulate multivariable equations. The chapter concludes with an introduction to complex imaginary numbers and direct variation problems, both frequently tested concepts.

Properties of Exponents and Powers

Covers: SAT Math Part 9 - Properties of Exponents and Powers - Membership (video, 11 min)

Exponents provide a shorthand for repeated multiplication. Understanding their properties is crucial for simplifying algebraic expressions on standardized tests.

Basic Definition of Exponents

An exponent indicates how many times a base number is multiplied by itself. For example, in a^n , 'a' is the base and 'n' is the exponent. This means 'a' is multiplied by itself 'n' times.

Key Exponent Properties

1. **Product Rule:** When multiplying terms with the same base, add the exponents.

- Formula: $a^m \cdot a^n = a^{(m+n)}$
- Example: $x^3 \cdot x^5 = x^{(3+5)} = x^8$

2. **Quotient Rule:** When dividing terms with the same base, subtract the exponents.

- Formula: $a^m / a^n = a^{(m-n)}$
- Example: $y^7 / y^2 = y^{(7-2)} = y^5$

3. **Power Rule:** When raising a power to another power, multiply the exponents.

- Formula: $(a^m)^n = a^{(m \cdot n)}$
- Example: $(z^4)^3 = z^{(4 \cdot 3)} = z^{12}$

4. **Zero Exponent Rule:** Any non-zero base raised to the power of zero is 1.

- Formula: $a^0 = 1$ (where $a \neq 0$)
- Example: $5^0 = 1$, $(3x)^0 = 1$

5. **Negative Exponent Rule:** A term with a negative exponent can be rewritten as its reciprocal with a positive exponent.

- Formula: $a^{-n} = 1 / a^n$
- Example: $x^{-3} = 1 / x^3$. Also, $1 / y^{-2} = y^2$.

6. Product to a Power Rule: When a product is raised to a power, apply the exponent to each factor.

- Formula: $(ab)^n = a^n * b^n$
- Example: $(2x)^3 = 2^3 * x^3 = 8x^3$

7. Quotient to a Power Rule: When a quotient is raised to a power, apply the exponent to both the numerator and the denominator.

- Formula: $(a/b)^n = a^n / b^n$
- Example: $(x/y)^2 = x^2 / y^2$

8. Fractional Exponents (Radicals): A fractional exponent indicates a root. The numerator is the power, and the denominator is the root.

- Formula: $a^{(m/n)} = (n\sqrt{a})^m = n\sqrt{(a^m)}$
- Example: $8^{(2/3)} = (3\sqrt{8})^2 = 2^2 = 4$

Worked Examples

Let us simplify the following expressions.

Example 1: Simplify $(x^2 * y^3)^4 / (x^5 * y^2)$

Step 1: Apply the power rule to the numerator. $(x^2 * y^3)^4 = x^{(2*4)} * y^{(3*4)} = x^8 * y^{12}$

Step 2: Rewrite the expression. $(x^8 * y^{12}) / (x^5 * y^2)$

Step 3: Apply the quotient rule for each base. For x: $x^8 / x^5 = x^{(8-5)} = x^3$ For y: $y^{12} / y^2 = y^{(12-2)} = y^{10}$

Step 4: Combine the simplified terms. The simplified expression is $x^3 * y^{10}$.

Example 2: Simplify $(3a^{-2} * b^3)^{-2}$

Step 1: Apply the power rule to all terms inside the parentheses. $(3^{-2} * (a^{-2})^{-2} * (b^3)^{-2})$ This becomes $3^{-2} * a^{(-2 * -2)} * b^{(3 * -2)}$ This simplifies to $3^{-2} * a^4 * b^{-6}$

Step 2: Apply the negative exponent rule to terms with negative exponents. $3^{-2} = 1 / 3^2 = 1/9$ $b^{-6} = 1 / b^6$

Step 3: Combine the terms. $(1/9) * a^4 * (1/b^6) = a^4 / (9b^6)$

Common Mistakes

- **Incorrectly distributing exponents:** $(a + b)^n$ is NOT equal to $a^n + b^n$. Exponents distribute over multiplication and division, not addition or subtraction.
- **Confusing negative base with negative exponent:** $-x^2$ is different from $(-x)^2$. $-x^2 = -(xx)$, while $(-x)^2 = (-x)(-x) = x^2$. A negative exponent means a reciprocal, not a negative number. For example, $2^{-3} = 1/8$, not -8.

- **Errors with fractional exponents:** Remember that $a^{(m/n)}$ means the n th root of a raised to the m th power.

Practice these rules regularly. Many test questions involve simplifying expressions using multiple exponent properties in combination.

Solving Absolute Value Equations and Linear Inequalities

Covers: Solving Absolute Value Equations - SAT Math Part 10 (video, 5 min) · SAT Math Part 11 - Linear Inequalities - Membership (video, 12 min)

This section covers two fundamental algebraic concepts: absolute value equations and linear inequalities. Both require careful attention to detail to avoid common pitfalls.

Solving Absolute Value Equations

The absolute value of a number is its distance from zero on the number line. For example, $|3| = 3$ and $|-3| = 3$. When solving an equation like $|x| = a$, it means x can be ' a ' or ' $-a$ '. This leads to two separate equations.

Procedure for Solving Absolute Value Equations:

1. **Isolate the absolute value expression:** Get the absolute value term by itself on one side of the equation.
2. **Set up two equations:** If $|\text{expression}| = c$ (where c is a positive number), then:
 - $\text{expression} = c$
 - $\text{expression} = -c$
3. **Solve each equation:** Find the value(s) of the variable.
4. **Check for extraneous solutions:** Substitute solutions back into the *original* equation. An absolute value cannot equal a negative number.

Example 1: Solve $|2x - 3| = 7$

Step 1: The absolute value expression is already isolated.

Step 2: Set up two equations. Equation 1: $2x - 3 = 7$ Equation 2: $2x - 3 = -7$

Step 3: Solve each equation. For Equation 1: $2x = 10$, so $x = 5$. For Equation 2: $2x = -4$, so $x = -2$.

Step 4: Check solutions. For $x = 5$: $|2(5) - 3| = |7| = 7$. (Correct) For $x = -2$: $|2(-2) - 3| = |-7| = 7$. (Correct)
The solutions are $x = 5$ and $x = -2$.

Example 2: Solve $|x + 4| + 1 = 0$

Step 1: Isolate the absolute value expression. $|x + 4| = -1$

Step 2: An absolute value expression cannot be equal to a negative number. Therefore, there is no solution to this equation.

Solving Linear Inequalities

Linear inequalities are similar to linear equations, but they use inequality symbols: $<$, $>$, $\leq$, or $\geq$.

Procedure for Solving Linear Inequalities:

1. **Treat it like an equation:** Use inverse operations to isolate the variable.
2. **Special Rule:** Multiplying or dividing both sides by a *negative* number reverses the direction of the inequality sign.
3. **Represent the solution:** Solutions can be written in inequality notation, interval notation, or graphed on a number line.

Example 3: Solve $3x - 5 < 10$

Step 1: Add 5 to both sides: $3x < 15$.

Step 2: Divide by 3: $x < 5$.

Step 3: The solution is $x < 5$. This can be graphed with an open circle at 5 and an arrow extending to the left.

Example 4: Solve $-2x + 1 \geq 7$

Step 1: Subtract 1 from both sides: $-2x \geq 6$.

Step 2: Divide by -2 and reverse the inequality sign: $x \leq -3$.

Step 3: The solution is $x \leq -3$. This can be graphed with a closed circle at -3 and an arrow extending to the left.

Compound Inequalities

These involve two inequalities joined by "and" or "or". "And" inequalities require the solution to satisfy both conditions, often written as $a < x < b$. "Or" inequalities satisfy at least one condition.

Example 5: Solve $-4 < 2x + 2 \leq 8$

Step 1: Subtract 2 from all parts: $-6 < 2x \leq 6$.

Step 2: Divide all parts by 2: $-3 < x \leq 3$.

Solution: The solution is $-3 < x \leq 3$.

Common Mistakes

- **Absolute Value:** Forgetting to set up two equations, or incorrectly solving when the absolute value equals a negative number.
- **Inequalities:** Failing to reverse the inequality sign when multiplying or dividing by a negative number.

Mastering these techniques is vital for a wide range of algebra problems on standardized tests.

Functions and Multivariable Equations

Covers: Evaluating Composite Functions - SAT Math Part 12 (video, 6 min) · SAT Math Part 13 - Composite Functions and Multivariable Equations - Membership (video, 10 min) · SAT Math Part 14 - Solving Equations With Fractions & Multivariable Expressions - Membership (video, 10 min)

Functions are a core concept in mathematics, representing a relationship where each input has exactly one output. This section explores evaluating composite functions and manipulating equations with multiple variables, often involving fractions.

Evaluating Functions

A function is often written as $f(x)$, where 'x' is the input and $f(x)$ is the output. To evaluate a function at a specific value, substitute that value for 'x' wherever it appears in the function's definition.

Example 1: Given $f(x) = 3x^2 - 2x + 1$, find $f(-2)$.

Substitute -2 for x: $f(-2) = 3(-2)^2 - 2(-2) + 1$ $f(-2) = 3(4) - (-4) + 1$ $f(-2) = 12 + 4 + 1$ $f(-2) = 17$

Composite Functions

A composite function is formed when one function is substituted into another. The notation $f(g(x))$ means to first evaluate the inner function $g(x)$, and then use that result as the input for the outer function $f(x)$. Similarly, $g(f(x))$ means to first evaluate $f(x)$, then use its result as the input for $g(x)$.

Procedure for Evaluating Composite Functions:

1. **Identify the inner function:** This is the function closest to the variable (e.g., $g(x)$ in $f(g(x))$).
2. **Evaluate the inner function first:** If given a numerical input, calculate $g(\text{number})$. If given a variable expression, substitute the expression into $g(x)$.
3. **Substitute the result into the outer function:** Use the output from step 2 as the input for the outer function $f(x)$.

Example 2: Given $f(x) = x^2 + 3$ and $g(x) = 2x - 1$, find $f(g(4))$.

Step 1: Evaluate the inner function $g(4)$. $g(4) = 2(4) - 1 = 8 - 1 = 7$

Step 2: Use the result (7) as the input for the outer function $f(x)$. $f(7) = 7^2 + 3 = 49 + 3 = 52$ So, $f(g(4)) = 52$.

Example 3: Given $f(x) = x^2 + 3$ and $g(x) = 2x - 1$, find $g(f(x))$.

Step 1: The inner function is $f(x) = x^2 + 3$.

Step 2: Substitute the expression for $f(x)$ into $g(x)$. $g(f(x)) = g(x^2 + 3)$ Wherever 'x' appears in $g(x)$, replace it with $(x^2 + 3)$. $g(x^2 + 3) = 2(x^2 + 3) - 1$

Step 3: Simplify the expression. $g(f(x)) = 2x^2 + 6 - 1$ $g(f(x)) = 2x^2 + 5$

Multivariable Equations and Solving with Fractions

Many equations on standardized tests involve multiple variables. Often, you need to solve for one variable in terms of the others. This requires using inverse operations to isolate the desired variable. Equations may also contain fractions, which can be cleared by multiplying by the least common denominator (LCD).

Procedure for Solving Multivariable Equations:

1. **Identify the variable to isolate:** Determine which variable you need to solve for.
2. **Use inverse operations:** Apply addition, subtraction, multiplication, or division to both sides of the equation to move terms away from the target variable.
3. **Factor if necessary:** If the target variable appears in multiple terms, factor it out to isolate it.
4. **Clear fractions:** If fractions are present, multiply every term in the equation by the LCD to eliminate them.

Example 4: Solve the formula $A = P(1 + rt)$ for t .

Step 1: The variable to isolate is 't'.

Step 2: Divide both sides by P . $A/P = 1 + rt$

Step 3: Subtract 1 from both sides. $A/P - 1 = rt$

Step 4: Divide both sides by r . $(A/P - 1) / r = t$ This can also be written as $t = (A - P) / (Pr)$ by finding a common denominator on the left side first.

Example 5: Solve $(x/2) + (y/3) = 1$ for y .

Step 1: The variable to isolate is 'y'.

Step 2: Clear the fractions by multiplying every term by the LCD, which is 6. $6 * (x/2) + 6 * (y/3) = 6 * 1$ $3x + 2y = 6$

Step 3: Subtract $3x$ from both sides. $2y = 6 - 3x$

Step 4: Divide by 2. $y = (6 - 3x) / 2$ This can also be written as $y = 3 - (3/2)x$.

Common Mistakes

- **Composite Functions:** Incorrectly evaluating $f(g(x))$ as $f(x) * g(x)$ or evaluating in the wrong order.
- **Multivariable Equations:** Algebraic errors when moving terms, especially with signs. Forgetting to multiply *every* term by the LCD when clearing fractions.

Consistent practice with these types of problems will build confidence and accuracy.

Complex Imaginary Numbers and Direct Variation

Covers: Simplifying Complex Imaginary Numbers - SAT Math Part 15 (video, 8 min) · Direct Variation Problems - SAT Math Part 16 (video, 5 min)

This section introduces two distinct but important topics: complex imaginary numbers, which extend the number system, and direct variation, a specific type of linear relationship. Both appear on standardized tests and require a clear understanding of their definitions and properties.

Complex Imaginary Numbers

The imaginary unit, denoted by i , is defined as the square root of -1 ($i = \sqrt{-1}$). This allows calculations with square roots of negative numbers. A **complex number** is expressed in the form $a + bi$, where 'a' and 'b' are real numbers, and 'i' is the imaginary unit. 'a' is the real part, and 'bi' is the imaginary part.

Powers of i

The powers of i follow a repeating cycle of four:

- $i^1 = i$
- $i^2 = -1$ (since $i = \sqrt{-1}$, then $i^2 = -1$)
- $i^3 = -i$
- $i^4 = 1$ To simplify i^n , divide n by 4 and use the remainder to determine its equivalent in the cycle (remainder 1 means i , 2 means -1 , 3 means $-i$, 0 means 1).

Example 1: Simplify i^{27} . Divide 27 by 4: $27 = 4 \cdot 6 + 3$. The remainder is 3. So, $i^{27} = i^3 = -i$.

Operations with Complex Numbers

Addition and Subtraction: Combine the real parts and the imaginary parts separately. $(a + bi) + (c + di) = (a + c) + (b + d)i$ $(a + bi) - (c + di) = (a - c) + (b - d)i$

Example 2: Simplify $(3 + 2i) + (1 - 5i)$. $(3 + 1) + (2 - 5)i = 4 - 3i$

Multiplication: Use the distributive property (FOIL method), remembering that $i^2 = -1$. $(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$

Example 3: Simplify $(2 + 3i)(4 - i)$. $8 - 2i + 12i - 3i^2 = 8 + 10i - 3(-1) = 8 + 10i + 3 = 11 + 10i$

Division (Rationalizing the Denominator): Multiply the numerator and denominator by the **complex conjugate** of the denominator. The conjugate of $(c + di)$ is $(c - di)$. This eliminates the imaginary part from the denominator.

Example 4: Simplify $(5 + 2i) / (1 - i)$. Multiply numerator and denominator by $(1 + i)$. Numerator: $(5 + 2i)(1 + i) = 5 + 5i + 2i + 2i^2 = 5 + 7i - 2 = 3 + 7i$ Denominator: $(1 - i)(1 + i) = 1^2 - i^2 = 1 - (-1) = 2$ Result: $(3 + 7i) / 2 = 3/2 + (7/2)i$

Direct Variation

Direct variation describes a relationship where one variable is a constant multiple of another. If y varies directly with x , the equation is:

- $y = kx$ where 'k' is the constant of proportionality. The graph of a direct variation is a straight line passing through the origin (0,0).

Procedure for Solving Direct Variation Problems:

1. **Set up the equation:** Write $y = kx$.
2. **Find the constant of proportionality (k):** Use the given pair of values for x and y to solve for k .
3. **Use k to find unknowns:** Substitute k back into $y = kx$ to find any other unknown value.

Example 5: If y varies directly with x , and $y = 12$ when $x = 4$, find y when $x = 7$.

Step 1: Set up the equation: $y = kx$.

Step 2: Find k using $y = 12$ and $x = 4$. $12 = k \cdot 4$ $k = 3$

Step 3: Use $k = 3$ to find y when $x = 7$. $y = 3 \cdot 7$ $y = 21$

So, when $x = 7$, $y = 21$.

Common Mistakes

- **Complex Numbers:** Forgetting that $i^2 = -1$. Algebraic errors during multiplication or division.
- **Direct Variation:** Confusing direct variation ($y = kx$) with other types of relationships.

Both complex numbers and direct variation are specific tools that expand your problem-solving capabilities for standardized tests.

KEY TAKEAWAYS

- Mastering exponent properties simplifies complex expressions.
- Absolute value equations require solving two separate cases.
- Multiplying or dividing an inequality by a negative number reverses the inequality sign.
- Composite functions involve evaluating one function within another.
- Solving multivariable equations means isolating one variable in terms of the others.
- Complex numbers extend the real number system by introducing i , the imaginary unit.
- Direct variation describes a linear relationship where one quantity is a constant multiple of another.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. Simplify the expression $(x^3 \cdot y^{-2})^4 / (x^2 \cdot y^5)$.
2. Solve the equation $|2x - 5| = 7$.
3. Solve the inequality $-3x + 4 < 10$ and graph the solution.
4. Given $f(x) = x^2 + 1$ and $g(x) = 3x - 2$, find $f(g(2))$.
5. Solve the equation $3x - 2y = 12$ for y in terms of x .
6. Simplify $(2 + 3i)(1 - i)$.

CHAPTER 3

SAT Math Review

This chapter strengthens your foundational algebra skills, which are critical for success on college admissions tests. We focus on manipulating algebraic expressions, understanding functions, and translating real-world scenarios into mathematical models. Mastering these concepts will improve your ability to solve complex problems efficiently and accurately.

You will learn to rearrange formulas, simplify intricate fractions, and perform polynomial division. We also cover the interpretation of functions and data through the lens of average rate of change. Finally, this chapter provides strategies for converting word problems into solvable equations and systems of inequalities, including specific techniques for consecutive integer problems.

Manipulating Algebraic Expressions and Formulas

Covers: Solving Formulas For a Specified Variable - SAT Math Part 17 · Simplifying Complex Fractions - SAT Math Part 20

Algebraic manipulation is a core skill for the SAT and ACT. Often, you will need to rearrange a formula to solve for a specific variable. This process involves applying inverse operations to both sides of an equation until the desired variable is isolated. Remember, whatever operation you perform on one side, you must perform the same operation on the other side to maintain equality.

Consider the formula for the area of a trapezoid: $A = (1/2)h(b_1 + b_2)$. To solve for h , the height, we need to undo the operations applied to h . First, multiply both sides by 2 to eliminate the fraction:

$$2A = h(b_1 + b_2)$$

Next, divide both sides by $(b_1 + b_2)$ to isolate h :

$$h = 2A / (b_1 + b_2)$$

This method applies to any variable in any formula. Always identify the operations acting on the target variable and apply their inverses in reverse order of operations.

Another crucial manipulation skill is simplifying complex fractions. A complex fraction is a fraction where the numerator, denominator, or both contain fractions. There are two primary strategies for simplification.

Method 1: Multiply by the reciprocal of the denominator. This is effective when the numerator and denominator are single fractions.

Example: Simplify $(x/y) / (z/w)$

$$(x/y) * (w/z) = xw / yz$$

Method 2: Multiply the numerator and denominator by the least common denominator (LCD) of all small fractions. This method is particularly useful when the numerator or denominator contains sums or differences of fractions.

Example: Simplify $((1/x) + 1) / ((1/x) - 1)$

Here, the small fractions are $1/x$. The LCD of $1/x$ and 1 is x . Multiply both the main numerator and main denominator by x :

$$\begin{aligned} & (x * ((1/x) + 1)) / (x * ((1/x) - 1)) \\ &= (x(1/x) + x(1)) / (x(1/x) - x(1)) \\ &= (1 + x) / (1 - x) \end{aligned}$$

Common mistakes include forgetting to distribute the LCD to every term in the numerator and denominator, or incorrectly applying inverse operations when solving for a variable. Practice these techniques with various formulas and complex fractions to build speed and accuracy.

Polynomials, Quadratic Functions, and Rates of Change

Covers: SAT Math Part 18 - Polynomials and Quadratic Functions · Long Division of Polynomials - SAT Math Part 19 · Average Rate of Change of Functions & Data Tables - SAT Math Part 21

Polynomials are expressions consisting of variables and coefficients, involving only the operations of addition, subtraction, multiplication, and non-negative integer exponents. A quadratic function is a specific type of polynomial of degree two, meaning the highest exponent of the variable is 2. Its general form is $f(x) = ax^2 + bx + c$, where a , b , and c are constants and a is not equal to zero. The graph of a quadratic function is a parabola, a U-shaped curve that opens either upward (if $a > 0$) or downward (if $a < 0$). Key features include the vertex (the turning point), the axis of symmetry, and the x-intercepts (roots or zeros).

Long division of polynomials is a method for dividing a polynomial by another polynomial of a lower or equal degree. It is similar to numerical long division. This process helps in factoring polynomials, finding roots, and simplifying rational expressions. Let's divide $x^3 - 2x^2 - 5x + 6$ by $x - 3$:

1. Divide the first term of the dividend (x^3) by the first term of the divisor (x) to get x^2 . Write x^2 above the x^2 term in the dividend.
2. Multiply x^2 by the entire divisor ($x - 3$) to get $x^3 - 3x^2$. Write this result below the dividend.
3. Subtract ($x^3 - 3x^2$) from ($x^3 - 2x^2$). Remember to distribute the negative sign: $(x^3 - 2x^2) - (x^3 - 3x^2) = x^2$.
4. Bring down the next term of the dividend ($-5x$). You now have $x^2 - 5x$.
5. Repeat the process: Divide the first term of $x^2 - 5x$ (x^2) by the first term of the divisor (x) to get x . Write x above the $-5x$ term.
6. Multiply x by ($x - 3$) to get $x^2 - 3x$. Write this below $x^2 - 5x$.
7. Subtract ($x^2 - 3x$) from ($x^2 - 5x$): $(x^2 - 5x) - (x^2 - 3x) = -2x$.

8. Bring down the last term of the dividend (+6). You now have $-2x + 6$.
9. Repeat: Divide $-2x$ by x to get -2 . Write -2 above the $+6$ term.
10. Multiply -2 by $(x - 3)$ to get $-2x + 6$. Write this below $-2x + 6$.
11. Subtract $(-2x + 6)$ from $(-2x + 6)$ to get 0 . The remainder is 0 .

The quotient is $x^2 + x - 2$. If there were a non-zero remainder, it would be written as a fraction over the divisor.

The average rate of change of a function over an interval $[a, b]$ is the slope of the secant line connecting the points $(a, f(a))$ and $(b, f(b))$. It is calculated using the formula: $(f(b) - f(a)) / (b - a)$. This concept extends to data tables. If you have a table of values, identify the y values corresponding to the x values a and b , then apply the same formula. For example, if a car travels 100 miles in 2 hours and 250 miles in 5 hours, the average rate of change (speed) between hours 2 and 5 is $(250 - 100) / (5 - 2) = 150 / 3 = 50$ miles per hour. This represents the average change in the dependent variable per unit change in the independent variable over the given interval.

Translating Word Problems into Equations and Inequalities

Covers: Writing Equations From Word Descriptions & Sentences - SAT Math Part 22 · Writing Systems of Inequalities - SAT Math Part 23 · SAT Math Part 24 - Consecutive Integers Word Problems - Membership

Many SAT and ACT math problems are presented as word problems. The first step to solving these is to translate the verbal descriptions into mathematical equations or inequalities. This requires careful reading and identifying keywords that indicate mathematical operations.

Keywords for Operations:

- **Addition:** sum, total, increased by, more than, plus
- **Subtraction:** difference, decreased by, less than, minus, fewer than
- **Multiplication:** product, times, of, twice, per
- **Division:** quotient, divided by, per, ratio
- **Equality:** is, was, will be, equals, results in

Start by defining your variables. For example, if a problem talks about "a number," assign it a variable like x . If it talks about "two numbers," use x and y or x and $x+1$ depending on their relationship. Then, construct the equation sentence by sentence.

Example: "The sum of three consecutive integers is 60." Let the first integer be x . Since they are consecutive, the next two are $x + 1$ and $x + 2$. The equation becomes: $x + (x + 1) + (x + 2) = 60$.

Consecutive integer problems are common. Remember these patterns:

- **Consecutive integers:** $x, x + 1, x + 2 \dots$
- **Consecutive even integers:** $x, x + 2, x + 4 \dots$ (where x is an even integer)
- **Consecutive odd integers:** $x, x + 2, x + 4 \dots$ (where x is an odd integer)

Writing systems of inequalities involves identifying constraints or conditions that limit possible values. Keywords like "at least," "at most," "no more than," "minimum," and "maximum" are crucial. "At least" translates to $\geq$ (greater than or equal to), while "at most" translates to $\leq$ (less than or equal to).

Example: A baker makes two types of cakes, A and B. Cake A requires 2 cups of flour and 1 cup of sugar. Cake B requires 3 cups of flour and 0.5 cups of sugar. The baker has 20 cups of flour and 8 cups of sugar available. Let a be the number of Cake A and b be the number of Cake B.

- **Flour constraint:** $2a + 3b \leq 20$ (Total flour used must be less than or equal to 20 cups)
- **Sugar constraint:** $1a + 0.5b \leq 8$ (Total sugar used must be less than or equal to 8 cups)
- **Non-negativity:** $a \geq 0$ and $b \geq 0$ (You cannot make a negative number of cakes)

This forms a system of inequalities. The solution set represents all possible combinations of cakes the baker can make given the ingredients. Always define variables clearly and ensure each constraint is represented by an appropriate inequality sign. Practice converting various real-world scenarios into mathematical expressions to improve your problem-solving skills.

KEY TAKEAWAYS

- Solving for a specified variable in a formula involves applying inverse operations to isolate the target variable.
- Complex fractions can be simplified by multiplying by the reciprocal of the denominator or by multiplying all terms by the least common denominator.
- Quadratic functions are degree-two polynomials whose graphs are parabolas, and polynomial long division helps simplify rational expressions or factor polynomials.
- The average rate of change measures how much a function's output changes on average per unit of input change over a given interval.
- Translating word problems into algebraic equations requires careful identification of keywords and clear definition of variables.
- Systems of inequalities represent multiple constraints in a problem, often using 'at least' for greater than or equal to, and 'at most' for less than or equal to.
- Consecutive integer problems follow specific patterns for setting up variables, such as x , $x+1$, $x+2$ for consecutive integers.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. How do you solve the formula $V = (1/3)\pi r^2 h$ for h ?
2. Describe two methods for simplifying complex fractions.
3. What is the average rate of change of $f(x) = x^2 + 1$ from $x = 1$ to $x = 3$?

4. How would you set up an equation for the problem: 'The product of two consecutive even integers is 48'?
5. A student needs to score at least 80% on a test to pass. If the test has 50 questions, write an inequality to represent the minimum number of questions the student must answer correctly.

CHAPTER 4

Advanced Problem Solving Techniques

This chapter builds on your foundational algebra and number skills to tackle more complex problem types seen on the SAT and ACT. You will learn to identify and solve problems involving geometric sequences and exponential growth, which describe quantities changing by a constant multiplier. The chapter also provides structured strategies for translating various word problems, including age, motion, mixture, ratio, and unit conversion scenarios, into solvable mathematical equations.

Finally, you will explore the principles of probability and counting, including permutations and combinations, which are essential for analyzing outcomes and likelihoods. Mastering these techniques will strengthen your ability to approach diverse math challenges with confidence and precision, preparing you for the varied question formats on college admissions tests.

Geometric Sequences and Exponential Growth

Covers: Geometric Sequences & Exponential Growth Problems - SAT Math Part 25

Geometric sequences describe a pattern where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio. This type of sequence models situations involving consistent growth or decay by a percentage or factor.

Understanding Geometric Sequences

A geometric sequence has the form $a, ar, ar^2, ar^3, \dots$ where a is the first term and r is the common ratio. To find the common ratio, divide any term by its preceding term. For example, in the sequence 2, 6, 18, 54, the common ratio r is $6/2 = 3$. The formula for the n -th term of a geometric sequence is:

$$a_n = a_1 * r^{(n-1)}$$

Here, a_n is the n -th term, a_1 is the first term, r is the common ratio, and n is the term number.

Example: Find the 6th term of the geometric sequence 3, 6, 12...

- First term $a_1 = 3$
- Common ratio $r = 6/3 = 2$
- We want the 6th term, so $n = 6$

Using the formula:

$$\begin{aligned} a_6 &= 3 * 2^{(6-1)} \\ a_6 &= 3 * 2^5 \\ a_6 &= 3 * 32 \\ a_6 &= 96 \end{aligned}$$

Connecting to Exponential Growth

Geometric sequences are a discrete form of exponential growth. Exponential growth is typically modeled by the formula:

$$y = a * b^x$$

Here, y is the final amount, a is the initial amount, b is the growth factor ($1 + \text{rate of growth}$), and x is the number of time periods. If the growth is by a percentage, the growth factor b is $(1 + \text{percentage rate as a decimal})$. For decay, it is $(1 - \text{percentage rate as a decimal})$.

Example: A city's population starts at 50,000 and grows by 4% annually. What will the population be after 3 years?

- Initial amount $a = 50,000$
- Growth rate $r = 0.04$ (4% as a decimal)
- Growth factor $b = 1 + 0.04 = 1.04$
- Number of years $x = 3$

Using the formula:

$$\begin{aligned}y &= 50000 * (1.04)^3 \\y &= 50000 * 1.124864 \\y &= 56243.2\end{aligned}$$

The population will be approximately 56,243 people after 3 years.

Common Mistakes and Practice

Students often confuse geometric sequences with arithmetic sequences (which have a common *difference*). Always check if terms are multiplied or added to get the next term. For exponential growth, ensure the rate is converted to a decimal and added to 1 for growth, or subtracted from 1 for decay. The exponent is typically the number of time periods, which corresponds to $n-1$ in the geometric sequence formula when n is the term number.

To practice, try creating your own geometric sequences and finding specific terms. Work through problems where a quantity increases or decreases by a fixed percentage over time, calculating the value after several periods.

Algebraic Word Problems: Age, Motion, and Mixtures

Covers: SAT Math Part 26 - Age Word Problems - Membership · Average Speed & Distance Rate Time Problems - SAT Math Part 27 · Mixture Problems (video, 15 min)

Many SAT and ACT math problems are presented as word problems, requiring you to translate verbal descriptions into algebraic equations. A systematic approach is crucial for these types of questions.

Age Word Problems

Age problems involve relationships between people's ages at different points in time. The key is to define variables clearly and account for changes in age over time.

Example: Sarah is currently three times as old as Tom. In 5 years, Sarah will be twice as old as Tom. How old is Tom now?

1. **Define variables:** Let Tom's current age be T . Sarah's current age is $3T$.
2. **Ages in 5 years:** Tom will be $T + 5$. Sarah will be $3T + 5$.
3. **Set up equation:** In 5 years, Sarah will be twice Tom's age: $3T + 5 = 2(T + 5)$.
4. **Solve:**

$$\begin{aligned}3T + 5 &= 2T + 10 \\3T - 2T &= 10 - 5 \\T &= 5\end{aligned}$$

Tom is currently 5 years old. Sarah is $3 * 5 = 15$ years old. In 5 years, Tom will be 10 and Sarah will be 20, which is twice Tom's age.

Distance, Rate, Time Problems

These problems use the fundamental formula $\text{Distance} = \text{Rate} \times \text{Time}$ ($D = RT$). They often involve objects moving towards each other, away from each other, or making a round trip.

Example: Two cars leave the same point at the same time, traveling in opposite directions. Car A travels at 50 mph, and Car B travels at 60 mph. How long will it take for them to be 440 miles apart?

1. **Define variables:** Let t be the time in hours.
2. **Distances:** Car A's distance $D_A = 50t$. Car B's distance $D_B = 60t$.
3. **Total distance:** Since they are moving in opposite directions, their distances add up to the total separation: $D_A + D_B = 440$.
4. **Set up equation:** $50t + 60t = 440$.
5. **Solve:**

$$\begin{aligned}110t &= 440 \\t &= 440 / 110 \\t &= 4\end{aligned}$$

It will take 4 hours for the cars to be 440 miles apart.

Mixture Problems

Mixture problems involve combining two or more quantities with different properties (like concentrations or costs) to create a new mixture. The key is to set up equations for both the total quantity and the total amount of the specific component being mixed.

Example: A chemist needs to mix a 20% acid solution with a 50% acid solution to produce 60 liters of a 30% acid solution. How many liters of each solution are needed?

1. **Define variables:** Let x be the liters of 20% solution. Let y be the liters of 50% solution.
2. **Total volume equation:** $x + y = 60$ (The total volume is 60 liters).
3. **Total acid equation:** The amount of acid from the first solution is $0.20x$. From the second, it is $0.50y$. The total acid in the final mixture is $0.30 * 60 = 18$ liters. So, $0.20x + 0.50y = 18$.
4. **Solve the system of equations:** From $x + y = 60$, we get $y = 60 - x$. Substitute this into the second equation:

$$\begin{aligned} 0.20x + 0.50(60 - x) &= 18 \\ 0.20x + 30 - 0.50x &= 18 \\ -0.30x &= 18 - 30 \\ -0.30x &= -12 \\ x &= -12 / -0.30 \\ x &= 40 \end{aligned}$$

If $x = 40$, then $y = 60 - 40 = 20$. The chemist needs 40 liters of the 20% solution and 20 liters of the 50% solution.

Common Mistakes and Practice

For all word problems, the most common error is misinterpreting the problem and setting up the wrong equations. Read carefully, define variables, and use tables or diagrams to organize information. For age problems, ensure all ages are adjusted correctly for past or future times. For motion problems, pay attention to relative speeds when objects are moving towards or away from each other. In mixture problems, remember to account for both the total quantity and the quantity of the specific component. Practice translating various scenarios into algebraic expressions and equations.

Ratios, Proportions, and Unit Conversions

Covers: SAT Math Part 28 - Unit Conversion Algebra Word Problems - Membership · SAT Math Part 29 - Ratios and Proportions Word Problems

Ratios, proportions, and unit conversions are fundamental tools for comparing quantities and changing measurement systems. They are frequently tested on college admissions exams.

Ratios and Proportions

A **ratio** compares two quantities. It can be written as $a:b$, a to b , or a/b . Ratios can also compare three or more quantities, such as $a:b:c$. A **proportion** is an equation stating that two ratios are equal. For example, $a/b = c/d$.

To solve a proportion for an unknown value, you can use cross-multiplication: if $a/b = c/d$, then $ad = bc$.

Example: If 5 apples cost \$3.00, how much do 8 apples cost?

1. **Set up a proportion:** We can compare apples to cost. $5 \text{ apples} / \$3.00 = 8 \text{ apples} / \x

2. **Cross-multiply:**

$$\begin{aligned} 5 * x &= 3.00 * 8 \\ 5x &= 24 \\ x &= 24 / 5 \\ x &= 4.80 \end{aligned}$$

8 apples cost \$4.80.

Unit Conversions

Unit conversion involves changing a measurement from one unit to another (e.g., feet to inches, hours to minutes). This is done using **conversion factors**, which are ratios equal to 1. For example, $12 \text{ inches} / 1 \text{ foot} = 1$.

The technique of **dimensional analysis** ensures that units cancel out correctly. You multiply the original measurement by one or more conversion factors until you reach the desired units.

Example: Convert 60 miles per hour to feet per second.

We need the following conversion factors:

- 1 mile = 5280 feet
- 1 hour = 60 minutes
- 1 minute = 60 seconds (so 1 hour = 3600 seconds)

Start with the given rate and multiply by conversion factors to cancel unwanted units and introduce desired ones:

$$(60 \text{ miles} / 1 \text{ hour}) * (5280 \text{ feet} / 1 \text{ mile}) * (1 \text{ hour} / 3600 \text{ seconds})$$

Notice how 'miles' and 'hours' units cancel out:

$$\begin{aligned} &= (60 * 5280 / 3600) \text{ feet} / \text{second} \\ &= (316800 / 3600) \text{ feet} / \text{second} \\ &= 88 \text{ feet} / \text{second} \end{aligned}$$

So, 60 miles per hour is 88 feet per second.

Complex Unit Conversions

Conversions can also involve squared or cubed units, such as square feet to square inches or cubic meters to cubic centimeters. In these cases, you must apply the conversion factor the appropriate number of times.

Example: Convert 2 square meters to square centimeters.

We know 1 meter = 100 centimeters. Therefore, 1 square meter = $(100 \text{ cm}) * (100 \text{ cm}) = 10,000 \text{ cm}^2$.

$$\begin{aligned} & 2 \text{ m}^2 * (100 \text{ cm} / 1 \text{ m}) * (100 \text{ cm} / 1 \text{ m}) \\ & = 2 * 100 * 100 \text{ cm}^2 \\ & = 20,000 \text{ cm}^2 \end{aligned}$$

Common Mistakes and Practice

For proportions, ensure that the units are consistent across the ratios. For example, if the numerator is 'apples', both numerators must be 'apples'. For unit conversions, the most common mistake is inverting a conversion factor or forgetting to apply it multiple times for squared or cubed units. Always write out the units and visually confirm they cancel correctly. Practice with various conversion problems, including those involving rates (like miles per hour) and area/volume units.

Probability and Counting Principles

Covers: Probability and Fractions - SAT Math Part 30 · SAT Math Part 31 - The Counting Principle, Permutations & Combinations - Membership

Probability and counting principles help us determine the likelihood of events and the number of ways certain outcomes can occur. These concepts are essential for data analysis questions.

Basic Probability

Probability is the measure of how likely an event is to occur. It is expressed as a number between 0 and 1, where 0 means impossible and 1 means certain. The formula for the probability of an event E is:

$$P(E) = (\text{Number of favorable outcomes}) / (\text{Total number of possible outcomes})$$

Example: What is the probability of rolling an even number on a standard six-sided die?

- Favorable outcomes (even numbers): 2, 4, 6 (3 outcomes)
- Total possible outcomes: 1, 2, 3, 4, 5, 6 (6 outcomes)

$$P(\text{even}) = 3 / 6 = 1/2$$

The Counting Principle

The Counting Principle states that if there are m ways to do one thing and n ways to do another, then there are $m * n$ ways to do both. This principle extends to any number of independent choices.

Example: A restaurant offers 3 appetizers, 5 main courses, and 2 desserts. How many different three-course meals can be ordered?

$$\begin{aligned} \text{Number of meals} &= (\text{Appetizers}) * (\text{Main Courses}) * (\text{Desserts}) \\ \text{Number of meals} &= 3 * 5 * 2 = 30 \end{aligned}$$

There are 30 different meal combinations.

Permutations

A **permutation** is an arrangement of objects in a specific order. The order of selection matters. The formula for the number of permutations of k objects chosen from a set of n distinct objects is:

$$P(n, k) = n! / (n - k)!$$

Here, $n!$ (n factorial) means $n * (n-1) * (n-2) * \dots * 1$.

Example: In a race with 8 runners, how many different ways can the gold, silver, and bronze medals be awarded?

- $n = 8$ (total runners)
- $k = 3$ (number of medals/positions)

$$\begin{aligned} P(8, 3) &= 8! / (8 - 3)! \\ P(8, 3) &= 8! / 5! \\ P(8, 3) &= (8 * 7 * 6 * 5 * 4 * 3 * 2 * 1) / (5 * 4 * 3 * 2 * 1) \\ P(8, 3) &= 8 * 7 * 6 \\ P(8, 3) &= 336 \end{aligned}$$

There are 336 ways to award the medals.

Combinations

A **combination** is a selection of objects where the order of selection does not matter. The formula for the number of combinations of k objects chosen from a set of n distinct objects is:

$$C(n, k) = n! / (k! * (n - k)!)$$

Example: From a group of 10 students, how many different committees of 3 students can be formed?

- $n = 10$ (total students)
- $k = 3$ (students on the committee)

$$\begin{aligned} C(10, 3) &= 10! / (3! * (10 - 3)!) \\ C(10, 3) &= 10! / (3! * 7!) \\ C(10, 3) &= (10 * 9 * 8 * 7!) / ((3 * 2 * 1) * 7!) \\ C(10, 3) &= (10 * 9 * 8) / (3 * 2 * 1) \\ C(10, 3) &= 720 / 6 \\ C(10, 3) &= 120 \end{aligned}$$

There are 120 different committees that can be formed.

Common Mistakes and Practice

The most frequent mistake in counting problems is confusing permutations and combinations.

Remember: if the order of selection or arrangement matters (like awarding medals or arranging books),

use permutations. If order does not matter (like forming a committee or choosing a hand of cards), use combinations. For probability, ensure you correctly identify both the number of favorable outcomes and the total possible outcomes. Practice by working through problems that clearly state whether order is important or not. If it is not explicitly stated, consider if changing the order of the selected items would result in a different outcome.

KEY TAKEAWAYS

- Geometric sequences model discrete exponential growth, characterized by a constant common ratio between consecutive terms.
- Algebraic word problems require careful translation of verbal information into mathematical equations by defining variables and setting up relationships.
- Distance, rate, time problems use the formula $D = RT$, while mixture problems often involve setting up equations based on total quantities and specific component amounts.
- Ratios compare quantities, proportions equate two ratios, and unit conversions use dimensional analysis with conversion factors to change measurement units.
- Basic probability is calculated as the ratio of favorable outcomes to the total number of possible outcomes.
- The Counting Principle multiplies the number of options for each stage to find total possibilities.
- Permutations count arrangements where order matters, while combinations count selections where order does not matter.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. How do you distinguish between a geometric sequence and an arithmetic sequence?
2. What is the primary strategy for solving word problems involving age, motion, or mixtures?
3. Explain how dimensional analysis helps in unit conversion problems.
4. When should you use a permutation versus a combination in a counting problem?

CHAPTER 5

SAT Math Review

This chapter builds on foundational arithmetic and algebra, introducing essential concepts frequently tested on standardized exams like the SAT and ACT. We will cover how to calculate and interpret averages, work with percentages for various real-world scenarios, and understand the basics of financial calculations like simple and compound interest. The chapter also revisits linear equations, focusing on slope and different forms of equations, which are critical for analyzing relationships between quantities.

Also, we will strengthen your geometry skills by exploring angles, lines, transversals, and the properties of triangles and quadrilaterals. A significant portion will be dedicated to similar triangles and proportions, a common topic that requires careful application of ratios. Mastering these topics will not only prepare you for specific test questions but also reinforce your overall mathematical reasoning.

Averages and Fundamental Percentages

Covers: Averages - SAT Math Part 32 (video, 15 min) · SAT Math Part 33 - Percents, Original Price, Sales Tax, & Discounts - Membership (video, 10 min) · SAT Math Part 36 - Percentages - Membership (video, 11 min)

The arithmetic mean, often simply called the **average**, is a fundamental concept. It is calculated by summing all the values in a set and then dividing by the number of values. For example, to find the average of 5, 10, and 15, you add them: $5 + 10 + 15 = 30$. Then divide by the count, which is 3: $30 / 3 = 10$. The average is 10.

Sometimes you might need to find a missing value given the average. If the average of four numbers is 8, and three of the numbers are 6, 9, and 7, what is the fourth number? The sum of the four numbers must be $4 * 8 = 32$. The sum of the known numbers is $6 + 9 + 7 = 22$. So, the fourth number is $32 - 22 = 10$.

Percentages represent a part of a whole, expressed as a fraction of 100. The word "percent" means "per hundred." To convert a percentage to a decimal, divide by 100 (move the decimal two places to the left). For example, 25% is 0.25. To convert a decimal to a percentage, multiply by 100 (move the decimal two places to the right). For instance, 0.75 is 75%. Fractions can also be converted to percentages by first converting to a decimal and then to a percentage. For example, $1/4$ is 0.25, which is 25%.

To find a percent of a number, convert the percentage to a decimal and multiply. To find 20% of 80, calculate $0.20 * 80 = 16$. If you need to find the whole when given a part and its percentage, divide the part by the decimal equivalent of the percentage. If 15 is 30% of a number, the number is $15 / 0.30 = 50$.

Percentages are used in many real-world applications, including **sales tax, discounts, and calculating original prices**. Sales tax is an amount added to the original price. If an item costs \$50 and there is a 6% sales tax, the tax is $0.06 * \$50 = \3 . The total cost is $\$50 + \$3 = \$53$. Alternatively, you can directly

calculate the total cost by multiplying the original price by $(1 + \text{tax rate})$: $\$50 * (1 + 0.06) = \$50 * 1.06 = \$53$.

Discounts are amounts subtracted from the original price. If an item costs \$80 and is discounted by 25%, the discount amount is $0.25 * \$80 = \20 . The sale price is $\$80 - \$20 = \$60$. Or, multiply the original price by $(1 - \text{discount rate})$: $\$80 * (1 - 0.25) = \$80 * 0.75 = \$60$.

Finding the **original price** after a discount can be tricky. If an item is on sale for \$75 after a 25% discount, it means \$75 represents $100\% - 25\% = 75\%$ of the original price. Let P be the original price. Then $0.75P = \$75$. To find P , divide \$75 by 0.75: $P = \$75 / 0.75 = \100 . Always check your work: a 25% discount on \$100 is \$25, making the sale price \$75. This matches the given information.

Common mistakes include forgetting to convert percentages to decimals before calculations, or incorrectly identifying the base amount when calculating sales tax or discounts. For original price problems, remember that the given price is a *percentage of the original*, not the original price itself.

Percent Change and Financial Applications

Covers: Percent Increase & Decrease, Population Growth Problems - SAT Math Part 34 (video, 10 min) · Simple interest and Compound Interest - SAT Math Part 35 (video, 13 min)

Percent increase and **percent decrease** measure the relative change in a quantity. The formula for percent change is: $(\text{New Value} - \text{Old Value}) / \text{Old Value} * 100\%$. If the result is positive, it is a percent increase. If it is negative, it is a percent decrease.

For example, if a population grew from 200 to 250, the percent increase is:

$$\begin{aligned} & ((250 - 200) / 200) * 100\% \\ & (50 / 200) * 100\% \\ & 0.25 * 100\% = 25\% \end{aligned}$$

If a price dropped from \$120 to \$90, the percent decrease is:

$$\begin{aligned} & ((90 - 120) / 120) * 100\% \\ & (-30 / 120) * 100\% \\ & -0.25 * 100\% = -25\% \end{aligned}$$

The negative sign indicates a decrease. When asked for a percent decrease, you typically report the positive value, such as "a 25% decrease."

Population growth problems often involve repeated percent increases. If a town's population of 10,000 increases by 5% each year, after one year it will be $10,000 * 1.05 = 10,500$. After two years, it will be $10,500 * 1.05 = 11,025$. This can be generalized using an exponential growth formula: $\text{Final Population} = \text{Initial Population} * (1 + \text{rate})^{\text{number of years}}$.

Simple interest is calculated only on the principal amount, the initial sum of money. The formula for simple interest is $I = PRT$, where:

- I = Interest earned

- P = Principal amount
- R = Annual interest rate (as a decimal)
- T = Time in years

If you invest \$1,000 at a simple interest rate of 4% per year for 3 years, the interest earned is:

$$I = \$1,000 * 0.04 * 3 = \$120$$

The total amount after 3 years would be Principal + Interest = \$1,000 + \$120 = \$1,120.

Compound interest is calculated on the principal amount and also on the accumulated interest from previous periods. This means your interest earns interest, leading to faster growth. The formula for compound interest is $A = P(1 + r/n)^{(nt)}$, where:

- A = Amount after time t
- P = Principal amount
- r = Annual interest rate (as a decimal)
- n = Number of times interest is compounded per year
- t = Time in years

Consider investing \$1,000 at an annual interest rate of 4% compounded annually (n=1) for 3 years:

$$\begin{aligned}A &= \$1,000 * (1 + 0.04/1)^{(1*3)} \\A &= \$1,000 * (1.04)^3 \\A &= \$1,000 * 1.124864 \\A &= \$1,124.86\end{aligned}$$

Notice that the compound interest yields a slightly higher amount (\$1,124.86) than simple interest (\$1,120) over the same period. The difference becomes much larger over longer periods or with higher interest rates. When solving problems, carefully read whether simple or compound interest is specified. A common error is using the initial value as the base for percent change calculations when the problem implies a cumulative change, or misapplying the compound interest formula components, especially 'n' for compounding frequency.

Linear Equations and Graphical Analysis

Covers: Slope and Linear Equations - SAT Math Part 37 (video, 11 min)

A **linear equation** is an algebraic equation that, when graphed, forms a straight line. These equations describe relationships where one quantity changes at a constant rate with respect to another. The most common forms are slope-intercept form, point-slope form, and standard form.

Slope is a measure of the steepness and direction of a line. It is defined as the "rise over run," or the change in the y-coordinates divided by the change in the x-coordinates between any two points on the line. Given two points (x₁, y₁) and (x₂, y₂), the slope (m) is calculated as:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

- A positive slope indicates the line rises from left to right.
- A negative slope indicates the line falls from left to right.
- A slope of zero (horizontal line) means y does not change.
- An undefined slope (vertical line) means x does not change.

For example, the slope of the line passing through (2, 3) and (5, 9) is:

$$m = (9 - 3) / (5 - 2) = 6 / 3 = 2$$

The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope and b is the y -intercept (the point where the line crosses the y -axis, (0, b)). This form is very useful for quickly graphing a line or identifying its key characteristics. If an equation is given as $2x + y = 5$, you can rewrite it as $y = -2x + 5$ to see that the slope is -2 and the y -intercept is 5 .

The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$. This form is particularly useful when you know the slope of a line and a single point (x_1, y_1) that it passes through. For instance, a line with a slope of 3 passing through the point (1, 2) would have the equation $y - 2 = 3(x - 1)$. You can then convert this to slope-intercept form: $y - 2 = 3x - 3$, so $y = 3x - 1$.

Standard form is typically written as $Ax + By = C$, where A , B , and C are integers, and A is usually positive. This form is often used for systems of equations. To convert from standard form to slope-intercept form, simply solve for y . For example, $3x + 4y = 12$ becomes $4y = -3x + 12$, then $y = (-3/4)x + 3$. Here, the slope is $-3/4$ and the y -intercept is 3 .

Parallel lines have the exact same slope. If line 1 has a slope of 2 , any line parallel to it will also have a slope of 2 .

Perpendicular lines have slopes that are negative reciprocals of each other. If a line has a slope of m , a line perpendicular to it will have a slope of $-1/m$. For example, if a line has a slope of 2 , a perpendicular line will have a slope of $-1/2$. If a line has a slope of $-3/4$, a perpendicular line will have a slope of $4/3$.

Common errors include mixing up the x and y coordinates when calculating slope, incorrectly applying the negative reciprocal rule for perpendicular lines, or making arithmetic mistakes when rearranging equations into different forms. Always double-check your calculations and the signs of your slopes.

Geometry: Angles, Lines, and Triangles

Covers: SAT Math Part 38 - Similar Triangles and Proportions - Membership (video, 8 min) · Angles, Lines, Transversals, Triangles & Quadrilaterals - SAT Math Part 39 (video, 10 min)

Geometry on the SAT and ACT often tests your understanding of basic shapes, angles, and their properties. **Angles** are formed by two rays sharing a common endpoint and are measured in degrees. Key types include acute (less than 90 degrees), right (exactly 90 degrees), obtuse (greater than 90 but less than 180 degrees), and straight (exactly 180 degrees). Pairs of angles can be complementary (sum to 90

degrees) or supplementary (sum to 180 degrees). **Vertical angles**, formed by intersecting lines, are always equal.

When two **parallel lines** are intersected by a **transversal**, specific angle relationships are formed. **Corresponding angles**, **alternate interior angles**, and **alternate exterior angles** are all equal. **Consecutive interior angles** (or same-side interior angles) are supplementary, meaning their sum is 180 degrees. Recognizing these relationships is crucial for solving geometry problems.

Triangles are three-sided polygons, and the sum of their interior angles is always 180 degrees. Common types include equilateral (all sides and angles equal), isosceles (two sides and their opposite angles equal), and scalene (all sides and angles different). **Right triangles** contain one 90-degree angle, and the **Pythagorean theorem** ($a^2 + b^2 = c^2$) applies, relating the lengths of the legs (a , b) to the hypotenuse (c).

Similar triangles have the same shape but not necessarily the same size. This means their corresponding angles are equal, and the ratio of their corresponding side lengths is constant. This property allows you to use **proportions** to find unknown side lengths. For example, if two similar triangles have corresponding sides in a ratio of 2:3, and one side in the smaller triangle is 4, its corresponding side in the larger triangle will be 6 (because $4/x = 2/3$, so $2x = 12$, $x = 6$).

Quadrilaterals are four-sided polygons whose interior angles sum to 360 degrees. Examples include squares, rectangles, parallelograms, rhombuses, and trapezoids, each with distinct properties regarding side lengths and angles. For instance, a rectangle is a quadrilateral with four right angles and opposite sides of equal length.

When working with similar triangles, always ensure you correctly identify corresponding sides and angles to set up proportions accurately. For parallel lines and transversals, carefully apply the angle relationships to find unknown measures. A common error is misidentifying which angles are equal or supplementary.

KEY TAKEAWAYS

- The arithmetic mean (average) is calculated by dividing the sum of values by the count of values.
- Percentages are fractions of 100 and are essential for calculating discounts, sales tax, and interest.
- Percent change measures the relative increase or decrease of a quantity and requires using the original value as the base.
- Simple interest is calculated only on the principal, while compound interest includes accumulated interest, leading to faster growth.
- The slope of a linear equation indicates its steepness and direction, and parallel and perpendicular lines have specific slope relationships.
- Understanding angle relationships formed by parallel lines and transversals is crucial for solving geometry problems.
- Similar triangles have equal corresponding angles and proportional corresponding side lengths, allowing for the use of ratios to find unknown measures.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. How do you calculate the average of a set of numbers, and what if you need to find a missing number when the average is known?
2. Explain the difference between simple interest and compound interest.
3. What is the slope-intercept form of a linear equation, and how can it be used to understand a line's characteristics?
4. Describe the angle relationships formed when two parallel lines are cut by a transversal.
5. What defines similar triangles, and how are proportions used with them?

CHAPTER 6

ACT Math Critical Concepts

This chapter is your comprehensive guide to the critical math concepts frequently tested on the ACT. It covers a broad spectrum of topics, from fundamental algebra to advanced geometry, trigonometry, and data analysis. The goal is to solidify your understanding of these areas, providing the foundational knowledge necessary for success on the exam.

Each section breaks down key principles, offers clear examples, and highlights important formulas. Studying this material will help you develop the problem-solving skills needed to approach even the most challenging ACT math questions with confidence.

Algebra and Advanced Mathematics

Covers: ACT Math: Critical Concepts for a 36 - Algebra (Part 1 of 4) · ACT Math: Critical Concepts for a 36 - Advanced Mathematics (Part 2 of 4)

This section focuses on core algebraic principles and advanced mathematical concepts frequently tested on the ACT. Mastering these topics is essential for solving a wide range of problems. We begin with a review of linear equations and inequalities, then move into systems of equations, functions, and quadratic expressions.

Linear Equations and Inequalities

Linear equations involve variables raised to the first power. To solve them, isolate the variable using inverse operations. For example, to solve $3x + 5 = 14$, subtract 5 from both sides to get $3x = 9$, then divide by 3 to find $x = 3$. Inequalities follow similar rules, but multiplying or dividing by a negative number reverses the inequality sign. Consider $-2x + 1 > 7$. Subtract 1: $-2x > 6$. Divide by -2 and reverse the sign: $x < -3$.

Systems of Equations

Systems of equations involve two or more equations with two or more variables. The goal is to find values for the variables that satisfy all equations simultaneously. Common methods include substitution and elimination.

- **Substitution:** Solve one equation for a variable, then substitute that expression into the other equation. Example: $y = 2x + 1$ $3x + y = 11$ Substitute $2x + 1$ for y in the second equation: $3x + (2x + 1) = 11$. Simplify: $5x + 1 = 11$. Solve for x : $5x = 10$, so $x = 2$. Substitute $x = 2$ back into $y = 2x + 1$: $y = 2(2) + 1 = 5$. The solution is $(2, 5)$.
- **Elimination:** Multiply one or both equations by a constant to make the coefficients of one variable opposites, then add the equations to eliminate that variable. Example: $2x + 3y = 7$ $4x - 3y = 5$ Add the equations directly: $(2x + 4x) + (3y - 3y) = 7 + 5$. $6x = 12$, so $x = 2$. Substitute $x = 2$ into the first equation: $2(2) + 3y = 7$. $4 + 3y = 7$, $3y = 3$, so $y = 1$. The solution is $(2, 1)$.

Functions

A function assigns each input (domain) exactly one output (range). Function notation, like $f(x)$, represents the output for a given input x .

- **Domain and Range:** The domain is the set of all possible input values for x . The range is the set of all possible output values for $f(x)$. For example, in $f(x) = \sqrt{x - 2}$, the domain requires $x - 2 \geq 0$, so $x \geq 2$. The range is $f(x) \geq 0$.
- **Types of Functions:**
- **Linear:** $f(x) = mx + b$ (straight line).
- **Quadratic:** $f(x) = ax^2 + bx + c$ (parabola).
- **Polynomial:** Functions with terms involving non-negative integer powers of x .
- **Rational:** Functions expressed as a ratio of two polynomials, like $f(x) = P(x) / Q(x)$.

Quadratic Equations

Quadratic equations are of the form $ax^2 + bx + c = 0$.

- **Factoring:** If $x^2 + 5x + 6 = 0$, factor into $(x + 2)(x + 3) = 0$. Solutions are $x = -2$ and $x = -3$.
- **Quadratic Formula:** For equations that do not factor easily, use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **Vertex Form:** $y = a(x - h)^2 + k$ where (h, k) is the vertex of the parabola.
- **Roots (Zeros):** The x -intercepts of the parabola, where $y = 0$. The discriminant, $b^2 - 4ac$, determines the nature of the roots:
 - > 0 : Two distinct real roots.
 - $= 0$: One real root (a double root).
 - < 0 : Two complex conjugate roots.

Exponents and Logarithms

Understand exponent rules: $x^a * x^b = x^{(a+b)}$, $(x^a)^b = x^{(ab)}$, $x^a / x^b = x^{(a-b)}$.

Logarithms are the inverse of exponentiation. If $b^y = x$, then $\log_b(x) = y$. Key properties include $\log_b(xy) = \log_b(x) + \log_b(y)$ and $\log_b(x^p) = p * \log_b(x)$.

Practice solving various types of equations and inequalities. Pay close attention to the conditions for domain restrictions in functions and the implications of the discriminant in quadratic equations.

Ratios, Percents, Data Analysis, and Probability

Covers: ACT Math: Critical Concepts for a 36 -Ratios, Percents, Data Analysis, and Probability (Part 3 of 4)

This section covers essential concepts in quantitative reasoning, including ratios, percentages, data interpretation, and probability. These topics are fundamental for understanding and solving real-world problems presented on the ACT.

Ratios and Proportions

A ratio compares two quantities. It can be written as $a:b$, a/b , or a to b . A proportion states that two ratios are equal. For example, if $2/3 = x/9$, cross-multiply to solve for x : $2 * 9 = 3 * x$, so $18 = 3x$, and $x = 6$.

- **Direct Variation:** Two quantities x and y vary directly if $y = kx$ for some constant k . As x increases, y increases proportionally.
- **Inverse Variation:** Two quantities x and y vary inversely if $y = k/x$ for some constant k . As x increases, y decreases.

Percentages

A percentage is a ratio out of 100. To convert a percent to a decimal, divide by 100. For example, 25% is 0.25.

- **Percent Increase/Decrease:** To find a percent change, use the formula: $((\text{New Value} - \text{Original Value}) / \text{Original Value}) * 100\%$. Example: A price increases from \$40 to \$50. The percent increase is $((50 - 40) / 40) * 100\% = (10 / 40) * 100\% = 0.25 * 100\% = 25\%$.
- **Finding a Part, Whole, or Percent:**
 - What is 20% of 80? $0.20 * 80 = 16$.
 - 25 is what percent of 200? $(25 / 200) * 100\% = 12.5\%$.
 - 30 is 15% of what number? $30 = 0.15 * x$, so $x = 30 / 0.15 = 200$.

Data Analysis

Data analysis questions often involve interpreting graphs, tables, and statistical measures.

- **Measures of Central Tendency:**
 - **Mean:** The average of a dataset (sum of values / number of values).
 - **Median:** The middle value when the data is ordered. If there is an even number of values, it is the average of the two middle values.
 - **Mode:** The value that appears most frequently. A dataset can have one mode, multiple modes, or no mode.
- **Measures of Spread:**
 - **Range:** The difference between the highest and lowest values.
 - **Standard Deviation:** A measure of how spread out the numbers are from the mean. A small standard deviation means data points are close to the mean. A large standard deviation means data points are spread out. You typically do not need to calculate standard deviation on the ACT but should understand its meaning.
 - **Interpreting Graphs:** Understand bar graphs, line graphs, pie charts, and scatter plots. Pay attention to labels, scales, and trends.

Probability

Probability is the likelihood of an event occurring. It is calculated as $\frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$. Probabilities are always between 0 (impossible) and 1 (certain).

- **Independent Events:** The outcome of one event does not affect the outcome of another. The probability of both occurring is $P(A \text{ and } B) = P(A) * P(B)$. Example: Rolling a 6 on a die and flipping heads on a coin. $(1/6) * (1/2) = 1/12$.
- **Dependent Events:** The outcome of the first event affects the outcome of the second. Example: Drawing two cards without replacement.
- **Combinations and Permutations:**
- **Permutations:** Order matters. $P(n, k) = n! / (n - k)!$
- **Combinations:** Order does not matter. $C(n, k) = n! / (k! * (n - k)!)$ The ACT usually tests simpler probability concepts. For complex counting problems, careful listing or logical reasoning often suffices.

Practice identifying the correct statistical measure for a given situation and correctly calculating probabilities for various scenarios. Focus on understanding the definitions and when to apply each concept.

Geometry, Trigonometry, Sequences, and Complex Numbers

Covers: ACT Math: Critical Concepts for a 36-Geometry, Trigonometry, Sequences and Comp Num (Part 4 of 4)

This final section brings together key concepts from geometry, trigonometry, sequences, and complex numbers. These areas often appear in the later, more challenging problems on the ACT Math test.

Geometry

Geometry questions on the ACT cover a broad range of topics.

- **Lines and Angles:** Understand parallel lines cut by a transversal (alternate interior, corresponding, consecutive interior angles). The sum of angles on a straight line is 180 degrees. Vertical angles are equal.
- **Triangles:**
- Sum of angles is 180 degrees.
- **Pythagorean Theorem:** For right triangles, $a^2 + b^2 = c^2$.
- **Special Right Triangles:**
- **45-45-90:** Sides are in ratio $x : x : x\sqrt{2}$.
- **30-60-90:** Sides are in ratio $x : x\sqrt{3} : 2x$.
- **Area:** $(1/2) * \text{base} * \text{height}$.
- **Quadrilaterals:** Understand properties of squares, rectangles, parallelograms, rhombuses, and trapezoids. Area formulas are important.

- **Circles:**
- **Circumference:** $C = 2\pi r$ or $C = \pi d$.
- **Area:** $A = \pi r^2$.
- **Arcs and Sectors:** Arc length is a fraction of the circumference, proportional to the central angle. Sector area is a fraction of the circle's area, proportional to the central angle.
- **Equations of a Circle:** $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius.
- **Solid Geometry:** Calculate volume and surface area for cubes, rectangular prisms, cylinders, cones, and spheres. Memorize key formulas.
- **Coordinate Geometry:** Distance formula, midpoint formula, slope of a line. Parallel lines have equal slopes. Perpendicular lines have slopes that are negative reciprocals.

Trigonometry

Trigonometry on the ACT primarily focuses on right-triangle trigonometry.

- **SOH CAH TOA:**
- $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$
- $\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$
- $\tan(\theta) = \text{Opposite} / \text{Adjacent}$
- **Unit Circle Basics:** Understand the signs of sine, cosine, and tangent in each quadrant. Know the values for common angles like 0, 30, 45, 60, 90 degrees (or 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$ radians).
- **Law of Sines:** $a / \sin(A) = b / \sin(B) = c / \sin(C)$ (for non-right triangles).
- **Law of Cosines:** $c^2 = a^2 + b^2 - 2ab\cos(C)$ (for non-right triangles). These laws are used to solve for unknown sides or angles in any triangle.

Sequences

- **Arithmetic Sequences:** Each term is found by adding a constant difference d to the previous term. The n th term is $a_n = a_1 + (n - 1)d$.
- **Geometric Sequences:** Each term is found by multiplying the previous term by a constant ratio r . The n th term is $a_n = a_1 * r^{(n - 1)}$.
- **Series:** The sum of the terms in a sequence. Formulas for the sum of the first n terms of arithmetic and geometric series are sometimes useful, but often the ACT tests pattern recognition or direct summation for a small number of terms.

Complex Numbers

Complex numbers extend the real number system by including the imaginary unit i , where $i^2 = -1$.

- **Form:** A complex number is written as $a + bi$, where a is the real part and b is the imaginary part.
- **Operations:**
- **Addition/Subtraction:** Combine real parts and imaginary parts separately. $(a + bi) + (c + di) = (a + c) + (b + d)i$.

- **Multiplication:** Use the distributive property (FOIL), remembering $i^2 = -1$. $(2 + 3i)(1 - i) = 2 - 2i + 3i - 3i^2 = 2 + i + 3 = 5 + i$.
- **Division:** Multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $c + di$ is $c - di$. Example: $(2 + 3i) / (1 - i) = ((2 + 3i)(1 + i)) / ((1 - i)(1 + i)) = (2 + 2i + 3i + 3i^2) / (1 - i^2) = (2 + 5i - 3) / (1 + 1) = (-1 + 5i) / 2 = -1/2 + (5/2)i$.
- **Powers of i :** $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$. The pattern repeats every four powers. To find i^n , divide n by 4 and use the remainder. For example, $i^{10} = i^{(4 \cdot 2 + 2)} = i^2 = -1$.

Regularly review geometric formulas and trigonometric relationships. Practice complex number operations until they become routine.

KEY TAKEAWAYS

- Mastering linear equations, systems, and quadratic functions is essential for a strong algebraic foundation.
- Understanding ratios, percentages, and data analysis measures allows for effective interpretation of quantitative information.
- Probability calculations require identifying favorable outcomes and the total possible outcomes, especially for independent and dependent events.
- A solid grasp of geometric formulas for area, volume, and coordinate geometry is crucial for many ACT problems.
- Trigonometric functions (SOH CAH TOA) and the Laws of Sines and Cosines are applied to solve for unknown sides and angles in triangles.
- Arithmetic and geometric sequences follow distinct patterns that help predict future terms or sums.
- Complex numbers extend the real number system, requiring specific rules for addition, subtraction, multiplication, and division involving the imaginary unit i .

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. How does multiplying or dividing an inequality by a negative number affect the inequality sign?
2. Explain the difference between a permutation and a combination.
3. What is the purpose of the discriminant in a quadratic equation?
4. Describe how to divide complex numbers.
5. What are the formulas for the area and circumference of a circle?

CHAPTER 7

Foundations Refresher

Chapter 7 provides a concise, high-impact review of the core algebra, geometry, and data analysis concepts most frequently tested on the SAT and ACT. This material is designed to solidify your fundamental skills, ensuring you can approach common problem types with speed and accuracy. Mastery of these topics is essential for building a strong foundation for any advanced test preparation.

Beyond reviewing key mathematical ideas, this chapter also guides you to the most effective, free, and official practice resources available. Understanding where and how to apply your refreshed skills is a critical step in your test preparation path, allowing you to transition smoothly from foundational learning to targeted practice.

Essential Math Concepts for Test Day

Covers: Linear Equations, the Test-Day Way · Geometry and Data Facts Worth Memorizing

The SAT and ACT heavily emphasize a core set of mathematical concepts. Mastering these fundamentals allows you to solve problems efficiently and build confidence. This section reviews the most critical algebraic, geometric, and data analysis facts.

Linear Equations: The Cornerstone

Linear relationships are the single most frequently tested math topic. Your goal is to make solving and interpreting them automatic.

Solving One-Variable Equations The primary rule for solving equations is simple: whatever operation you perform on one side of the equation, you must perform on the other. Your objective is to isolate the variable.

Consider this example:

$$3(x - 2) + 5 = 2x + 9$$

First, distribute the 3 on the left side:

$$3x - 6 + 5 = 2x + 9$$

Combine like terms:

$$3x - 1 = 2x + 9$$

Gather the variable terms on one side by subtracting $2x$ from both sides:

$$x - 1 = 9$$

Add 1 to both sides to isolate x:

$$x = 10$$

Always take ten seconds to check your answer by substituting it back into the original equation. This simple habit catches most sign errors.

Slope: Rate and Steepness Slope measures a line's steepness. It is calculated as the 'rise over run' or the change in y divided by the change in x: **slope = $(y_2 - y_1) / (x_2 - x_1)$** .

Tests frequently use slope in several ways:

- In the slope-intercept form, $y = mx + b$, 'm' is the slope, and 'b' is the y-intercept (the point where the line crosses the y-axis).
- In word problems, slope represents a rate, such as cost per item, miles per hour, or dollars per month. The y-intercept represents a starting value, like an initial fee or a base cost. For instance, 'a gym charges 25 dollars to join plus 10 dollars per month' translates to $y = 10x + 25$.
- Parallel lines always have the same slope. Perpendicular lines have slopes that multiply to -1.

Systems of Equations A system of equations involves two or more equations with the same variables. The solution is the (x, y) pair that satisfies all equations simultaneously. Common methods include substitution (solving one equation for a variable and plugging it into the other) and elimination (adding or subtracting equations to cancel a variable). For multiple-choice questions, testing the given answer choices in both equations is often the fastest approach.

Special cases to remember: If two lines are parallel (same slope, different y-intercepts), there is no solution. If the two equations represent the exact same line, there are infinitely many solutions.

Inequalities: The Sign Flip Solving inequalities is similar to solving equations, with one crucial difference. If you multiply or divide both sides of an inequality by a negative number, you must reverse the inequality sign. Forgetting this rule is a common trap on standardized tests.

Geometry and Data Facts

These topics draw from a compact set of facts. Memorizing them will save you valuable time.

Angles and Triangles

- Angles on a straight line sum to 180 degrees. Angles around a point sum to 360 degrees. Vertical angles (angles opposite each other where two lines intersect) are equal.
- The angles inside any triangle sum to 180 degrees. An exterior angle of a triangle equals the sum of its two remote interior angles.
- **Pythagorean Theorem:** For a right triangle, $a^2 + b^2 = c^2$, where 'c' is the hypotenuse. Memorize common Pythagorean triples: 3-4-5, 5-12-13, and their multiples (for example, 6-8-10). These appear frequently.

- **Special Right Triangles:**

- 45-45-90 triangle: side ratios are 1 : 1 : root 2.
- 30-60-90 triangle: side ratios are 1 : root 3 : 2. These ratios often replace complex trigonometry on the tests.
- Similar triangles have equal angles and proportional sides. This principle is key to solving problems involving scaled figures or indirect measurement.

Circles

- Area = πr^2
- Circumference = $2 \pi r$
- A sector (slice) of a circle, or an arc (part of the circumference), scales proportionally to its central angle. For example, a central angle of 90 degrees (one-fourth of 360) corresponds to one-fourth of the circle's area and one-fourth of its circumference.

Averages, Percents, and Probability

- **Averages:** The formula is $\text{Average} = \text{Total} / \text{Count}$. The most powerful application is working backward: $\text{Total} = \text{Average} \times \text{Count}$. For example, if a student averages 85 on four tests, their total score is $85 \times 4 = 340$. If they need an average of 87 on five tests, the new total must be $87 \times 5 = 435$. They need to score $435 - 340 = 95$ on the fifth test.
- The median is the middle value in a sorted data set and is less affected by outliers than the mean (average).
- **Percents:** Percent change is calculated as $(\text{Change} / \text{Original Amount}) \times 100\%$.
- **Probability:** Probability = $(\text{Favorable Outcomes} / \text{Total Outcomes})$. Probabilities are always between 0 and 1 (or 0% and 100%). For data tables, probability questions usually require careful reading of rows and columns.

To solidify these facts, hand-copying them once and then quizzing yourself weekly is highly effective. This converts derivation time into quick recall on test day.

Your Official Practice Arsenal

Covers: Your Official (Free) Practice Arsenal

Once you have rebuilt your foundational math skills, the next crucial step is to apply them to authentic test questions. Using official, free resources ensures that your practice accurately reflects the style, format, and content of the actual SAT and ACT exams. This section outlines the best places to find these materials and suggests a strategic approach to your preparation.

Official Resources for the SAT

- **Bluebook:** This is the College Board's official testing application. It provides full-length, adaptive Digital SAT practice tests. You will need a free account to access it. It is best to take your first full

practice test only after you have completed this algebra refresher. A diagnostic test taken too early will primarily measure the knowledge gaps you are already working to close.

- **Khan Academy Official Digital SAT Prep:** This free, official, and personalized platform offers skill practice tailored for the Digital SAT. Its math skill drills are a natural progression after reviewing each topic in this course. You should work through these drills directly on their site.
- **College Board Question Bank:** This resource provides official questions that you can filter by domain and difficulty. It is an excellent tool for targeted practice on specific areas where you need more work.

Official Resources for the ACT

- **ACT.org Preparation Hub:** The official ACT website offers free practice questions and up-to-date information about the test. While there, make sure to check the current format of the enhanced ACT for your specific test date.
- **Official Free Practice Forms:** ACT also provides downloadable official practice tests with answer keys. Always use the most current forms available on their preparation pages to ensure relevance.

Sequencing Your Preparation

Follow this four-step sequence to maximize the effectiveness of your study time:

1. **Foundations First (This Course):** Before you work through advanced strategies, ensure your fundamental skills are solid. Solving linear equations, applying the Pythagorean theorem, and reverse-engineering averages must be automatic. Attempting strategy courses without this base is like building on sand.
2. **Then a Diagnostic Test:** Take one official practice test for the exam you are leaning towards. If you are undecided between the SAT and ACT, take one of each. This provides objective data to help you choose the best test for you and identifies specific areas for improvement.
3. **Then Strategy:** After your diagnostic, move to a test-specific course. MegaCourse offers options like 'Digital SAT Math Mastery' and 'ACT Complete Prep'. During this phase, focus on drilling official questions by topic, applying the strategies you learn.
4. **Always Be Closing Gaps:** Test preparation is an iterative process. Whenever a problem from a strategy course reveals a weakness in your foundational understanding, revisit the relevant video or lesson in this course within the same week. Consistent review, even ten minutes of mixed problems daily, is crucial to prevent your foundational skills from decaying.

KEY TAKEAWAYS

- Linear equations are the most frequently tested math topic; master solving them, interpreting slope as a rate, and understanding systems of equations.
- Remember the golden rule of equations: whatever you do to one side, do to the other, and always check your solution.
- When solving inequalities, reverse the sign if you multiply or divide both sides by a negative number.
- Memorize key geometry facts for angles, triangles (including Pythagorean triples and special right triangles), and circle formulas.
- Use the average formula backward ($\text{Total} = \text{Average} \times \text{Count}$) to solve many complex average problems efficiently.
- Practice exclusively with official, free SAT and ACT resources to ensure your preparation aligns with the actual tests.
- Follow a structured prep sequence: solidify foundations, take a diagnostic, learn test strategies, and continuously review foundational concepts.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. What is the crucial rule to remember when solving inequalities that differs from solving equations?
2. How can you quickly determine if two lines are parallel or perpendicular based on their slopes?
3. Explain the 'power move' for solving average problems on tests.
4. List two common Pythagorean triples and two special right triangle side ratios.
5. Why is it important to use official practice materials for SAT and ACT preparation?

CHAPTER 8

Final Assessment: Math Foundations

This chapter marks the culmination of your work in the "SAT & ACT Math Foundations" course. Module 8 provides a final assessment designed to test your mastery of the core algebra, geometry, and data analysis skills covered throughout the previous modules. Think of this assessment as a crucial diagnostic tool, not just a final grade.

Your performance here will highlight areas where your understanding is solid and pinpoint specific topics that may require further review. This comprehensive check ensures you have a strong mathematical foundation, preparing you effectively for any advanced SAT or ACT strategy course.

What You Should Have Mastered

Covers: Final Assessment: Math Foundations (quiz, 5 min)

By this point, you should have a solid grasp of the fundamental math concepts essential for college admissions tests. This includes rebuilding your understanding of **linear equations**, **systems of equations**, **exponents**, and **quadratics**. You should also be comfortable with key facts from **geometry** and **data analysis** that frequently appear on these exams.

Beyond specific topics, you should have developed effective problem-solving habits. This means knowing when to use a calculator efficiently and when to solve problems without one. The final assessment evaluates your ability to apply these skills across various problem types, ensuring your foundation is robust enough for advanced test preparation.

How to Approach the Final Assessment

Covers: Final Assessment: Math Foundations (quiz, 5 min)

Treat this assessment seriously, as if it were a timed section of a real SAT or ACT. Find a quiet environment where you can focus without interruption. Even for a short quiz, practicing good test-taking habits is valuable.

Read Carefully

Always read each question completely before attempting to solve it. Pay close attention to keywords, units, and what the question is specifically asking for. A common error is solving for the wrong variable or providing a different piece of information than requested.

Show Your Work

Even on a multiple-choice quiz, writing out your steps helps prevent careless errors. It also makes it easier to review your work later if you get a question wrong. If you are stuck, try to eliminate incorrect answer choices. This increases your probability of selecting the correct answer if you must guess.

Common Pitfalls and How to Avoid Them

Covers: Final Assessment: Math Foundations (quiz, 5 min)

Students often stumble on common types of errors, even when they understand the underlying math concepts. Being aware of these can help you avoid them.

- **Careless Arithmetic Errors:** Double-check your calculations, especially with negative numbers or fractions. A simple addition or subtraction mistake can lead to an incorrect answer.
- **Misinterpreting Word Problems:** Break down word problems into smaller parts. Identify the knowns and unknowns. Translate the words into mathematical equations or expressions carefully.
- **Forgetting Formulas or Properties:** Review key formulas for geometry, exponent rules, and quadratic properties. Practice applying them so they become second nature.
- **Rushing:** Take your time. A short quiz still allows for careful thought. Rushing often leads to mistakes that could have been avoided with a moment more consideration.

Interpreting Your Results

Covers: Final Assessment: Math Foundations (quiz, 5 min)

Your score on the Final Assessment is just one piece of information. The most valuable part of this assessment comes from reviewing your answers. Do not simply look at your score and move on.

Analyze Your Mistakes

Go through every question you answered incorrectly. For each one, determine *why* you got it wrong. Was it a conceptual misunderstanding? A careless error? Did you misread the question? Did you forget a formula?

Identify Weak Areas

Look for patterns in your incorrect answers. If you missed several geometry questions, that indicates geometry is an area for further study. If you struggled with quadratic equations, revisit that module. Use your results to create a targeted study plan. Go back to the specific lessons or chapters in this guide that cover those topics. Re-do practice problems and seek out additional exercises until you feel confident in those areas. This diagnostic approach will make your future study much more efficient.

KEY TAKEAWAYS

- The Final Assessment tests your mastery of all core math skills from this course.
- Approach the assessment like a real test, focusing on careful reading and showing your work.
- Common mistakes include careless arithmetic, misinterpreting word problems, and forgetting formulas.
- Review every question, especially incorrect ones, to understand the source of your errors.
- Use your assessment results to identify specific weak areas and guide your future study.
- A strong foundation from this course is essential for success in any SAT or ACT strategy program.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. What is the primary purpose of the Final Assessment in this course?
2. How should you prepare for and approach this assessment?
3. What are some common mistakes students make on math assessments?
4. After completing the assessment, what is the most important step to take?
5. Why is it important to rebuild math foundations before an SAT or ACT strategy course?

REFERENCE

Glossary

Absolute value

The distance of a number from zero on the number line, always non-negative.

Algebraic expression

A mathematical phrase that can contain numbers, variables, and operation symbols, but no equality sign.

Arithmetic mean

The sum of a set of numbers divided by the count of the numbers in the set, commonly known as the average.

Arithmetic sequence

A sequence of numbers where the difference between consecutive terms is constant.

Average rate of change

The slope of the secant line between two points on a function's graph, calculated as $(f(b) - f(a)) / (b - a)$.

Combination

A selection of objects from a larger set where the order of selection does not matter.

Complex fraction

A fraction in which the numerator, denominator, or both contain fractions.

Complex number

A number that can be expressed in the form $a + bi$, where 'a' and 'b' are real numbers and 'i' is the imaginary unit.

Composite function

A function formed by applying one function to the result of another function.

Compound interest

Interest calculated on the initial principal and also on the accumulated interest from previous periods.

Consecutive integers

Integers that follow each other in order, differing by 1 (e.g., x , $x+1$, $x+2$).

Constant of proportionality

The constant 'k' in a direct variation equation ($y = kx$), representing the ratio between the two variables.

Counting principle

A method to find the total number of possible outcomes for a sequence of events by multiplying the number of ways each event can occur.

Data analysis

The process of inspecting, cleaning, transforming, and modeling data with the goal of discovering useful information, drawing conclusions, and supporting decision-making.

Diagnostic assessment

A type of test designed to identify a student's existing knowledge, skills, and areas of difficulty in a specific subject.

Direct variation

A relationship where one variable is a constant multiple of another, expressed as $y = kx$.

Discriminant

The expression $b^2 - 4ac$ from the quadratic formula, which determines the number and type of roots of a quadratic equation.

Equation

A statement that two algebraic expressions are equal.

Exponent

A number indicating how many times a base number is multiplied by itself.

Exponential growth

A pattern of increase where the rate of growth is proportional to the current amount, often modeled by $y = a * b^x$.

Exponents

A number indicating how many times a base number is multiplied by itself, written as a superscript.

Extraneous solution

A solution derived from an equation that does not satisfy the original equation, often arising in rational or radical equations.

Factoring

The process of breaking down an expression into a product of simpler expressions or terms.

Function

A relationship where each input (x-value) corresponds to exactly one output (y-value).

Geometric sequence

A sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio.

Geometry

The branch of mathematics concerned with the properties and relations of points, lines, surfaces, solids, and higher dimensional analogs.

Hypotenuse

The longest side of a right triangle, opposite the right angle.

Imaginary unit (i)

Defined as the square root of -1, allowing for calculations with square roots of negative numbers.

Inequality

A mathematical statement comparing two expressions using symbols like $<$, $>$, $\leq$, or $\geq$.

Linear equations

Algebraic equations whose graph is a straight line, typically in the form $y = mx + b$.

Median

The middle value in a dataset when the values are arranged in order, or the average of the two middle values for an even number of data points.

Multivariable equation

An equation containing two or more variables.

Percent change

The ratio of the change in a value to its original value, expressed as a percentage. Calculated as $(\text{Change} / \text{Original Amount}) \times 100\%$.

Percentage

A way to express a number as a fraction of 100, denoted by the percent sign (%).

Permutation

An arrangement of objects in a specific order, where the sequence or order of selection is significant.

Polynomial

An expression consisting of variables and coefficients, involving only addition, subtraction, multiplication, and non-negative integer exponents of variables.

Probability

A numerical measure of the likelihood that an event will occur, expressed as a fraction or decimal between 0 and 1.

Proportion

An equation that states that two ratios are equal, commonly solved using cross-multiplication.

Pythagorean theorem

A fundamental relation in Euclidean geometry among the three sides of a right triangle: $a^2 + b^2 = c^2$, where 'c' is the hypotenuse.

Quadratic equation

An equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and a is not zero.

Quadratic function

A polynomial function of degree two, generally written as $f(x) = ax^2 + bx + c$ where a is not zero.

Quadratics

Polynomial equations of the second degree, often involving a variable raised to the power of two, such as $ax^2 + bx + c = 0$.

Similar triangles

Triangles that have the same shape but potentially different sizes, meaning their corresponding angles are equal and their corresponding sides are proportional.

Simple interest

Interest calculated only on the initial principal amount of a loan or deposit.

Slope

A measure of the steepness and direction of a line, calculated as the ratio of the vertical change (rise) to the horizontal change (run) between two points.

Soh cah toa

An acronym used to remember the definitions of the sine, cosine, and tangent ratios in right-angle trigonometry.

Standard deviation

A measure of the average distance between each data point and the mean of the dataset, indicating the spread of the data.

System of equations

A set of two or more equations containing two or more variables, for which a common solution is sought.

System of inequalities

A set of two or more inequalities with the same variables, whose solution is the region satisfying all inequalities simultaneously.

Systems of equations

A set of two or more equations with the same variables that are solved simultaneously to find common solutions.

Transversal

A line that intersects two or more other lines, often parallel lines, creating various angle relationships.

Unit conversion

The process of changing a measurement from one unit to another using conversion factors and dimensional analysis.

Y-intercept

The point where a line crosses the y-axis. In the equation $y = mx + b$, 'b' represents the y-intercept, often signifying a starting value.

REFERENCE

Answer Key

Chapter 1: Algebra Foundations for SAT Math

1. If $f(x) = x^2 - 3x + 1$, what is the value of $f(-2)$?

To find $f(-2)$, substitute $x = -2$ into the function: $f(-2) = (-2)^2 - 3(-2) + 1$. This simplifies to $4 + 6 + 1$, resulting in 11.

2. For what value of x is the expression $(x + 5) / (2x - 8)$ undefined?

An expression is undefined when its denominator equals zero. Set $2x - 8 = 0$. Solving for x gives $2x = 8$, so $x = 4$. The expression is undefined when $x = 4$.

3. Solve the quadratic equation $x^2 - 7x + 10 = 0$ by factoring.

Factor the trinomial into two binomials: $(x - 2)(x - 5) = 0$. Using the Zero Product Property, set each factor to zero: $x - 2 = 0$ or $x - 5 = 0$. This gives the solutions $x = 2$ and $x = 5$.

4. Solve the radical equation $\sqrt{3x - 5} = 4$.

First, square both sides of the equation to eliminate the radical: $(\sqrt{3x - 5})^2 = 4^2$, which simplifies to $3x - 5 = 16$. Then, solve for x : $3x = 21$, so $x = 7$. Check the solution: $\sqrt{3(7) - 5} = \sqrt{21 - 5} = \sqrt{16} = 4$. The solution $x = 7$ is valid.

5. Use the elimination method to solve the system of equations: $2x + y = 10$ and $x - y = 2$.

Add the two equations together. The y terms will cancel out: $(2x + y) + (x - y) = 10 + 2$, which simplifies to $3x = 12$. Solving for x gives $x = 4$. Substitute $x = 4$ into the second equation: $4 - y = 2$, which means $y = 2$. The solution to the system is $(4, 2)$.

Chapter 2: Algebra Foundations: Exponents, Functions, and Special Equations

1. Simplify the expression $(x^3 * y^{-2})^4 / (x^2 * y^5)$.

First, apply the power rule to the numerator: $(x^3 * y^{-2})^4$ becomes $x^{12} * y^{-8}$. Then, divide by the denominator $(x^2 * y^5)$ using the quotient rule. For x , $x^{12} / x^2 = x^{10}$. For y , $y^{-8} / y^5 = y^{-(8-5)} = y^{-3}$. The simplified expression is x^{10} / y^3 .

2. Solve the equation $|2x - 5| = 7$.

Set up two separate equations: $2x - 5 = 7$ and $2x - 5 = -7$. Solving the first equation gives $2x = 12$, so $x = 6$. Solving the second equation gives $2x = -2$, so $x = -1$. Both solutions are valid after checking.

3. Solve the inequality $-3x + 4 < 10$ and graph the solution.

Subtract 4 from both sides to get $-3x < 6$. Then, divide both sides by -3 . Remember to reverse the inequality sign when dividing by a negative number, resulting in $x > -2$. The solution is an open circle at -2 with an arrow extending to the right on a number line.

4. Given $f(x) = x^2 + 1$ and $g(x) = 3x - 2$, find $f(g(2))$.

First, evaluate the inner function $g(2)$: $g(2) = 3(2) - 2 = 6 - 2 = 4$. Next, use this result as the input for $f(x)$: $f(4) = 4^2 + 1 = 16 + 1 = 17$. So, $f(g(2)) = 17$.

5. Solve the equation $3x - 2y = 12$ for y in terms of x .

Subtract $3x$ from both sides to get $-2y = 12 - 3x$. Then, divide both sides by -2 to isolate y . This gives $y = (12 - 3x) / -2$, which simplifies to $y = -6 + (3/2)x$.

6. Simplify $(2 + 3i)(1 - i)$.

Use the FOIL method: $(2)(1) + (2)(-i) + (3i)(1) + (3i)(-i)$. This simplifies to $2 - 2i + 3i - 3i^2$. Since $i^2 = -1$, substitute to get $2 + i - 3(-1) = 2 + i + 3 = 5 + i$. The simplified complex number is $5 + i$.

Chapter 3: SAT Math Review**1. How do you solve the formula $V = (1/3)\pi r^2 h$ for h ?**

To solve $V = (1/3)\pi r^2 h$ for h , first multiply both sides by 3 to clear the fraction, resulting in $3V = \pi r^2 h$. Then, divide both sides by πr^2 to isolate h . The final formula is $h = 3V / (\pi r^2)$.

2. Describe two methods for simplifying complex fractions.

One method is to multiply the numerator by the reciprocal of the denominator, which works well for simple complex fractions. The second method, suitable for more complex expressions, involves finding the least common denominator (LCD) of all small fractions within the complex fraction. Then, multiply both the main numerator and the main denominator by this LCD.

3. What is the average rate of change of $f(x) = x^2 + 1$ from $x = 1$ to $x = 3$?

First, find $f(1)$ and $f(3)$. $f(1) = 1^2 + 1 = 2$. $f(3) = 3^2 + 1 = 10$. Using the formula $(f(b) - f(a)) / (b - a)$, the average rate of change is $(10 - 2) / (3 - 1) = 8 / 2 = 4$.

4. How would you set up an equation for the problem: 'The product of two consecutive even integers is 48'?

Let the first even integer be x . Since the integers are consecutive and even, the next even integer is $x + 2$. The product is 48, so the equation is $x(x + 2) = 48$. This expands to a quadratic equation, $x^2 + 2x - 48 = 0$.

5. A student needs to score at least 80% on a test to pass. If the test has 50 questions, write an inequality to represent the minimum number of questions the student must answer correctly.

Let c be the number of questions answered correctly. To score at least 80%, the proportion of correct answers must be $c/50 \geq 0.80$. Multiplying by 50, we get $c \geq 40$. The student must answer at least 40 questions correctly.

Chapter 4: Advanced Problem Solving Techniques**1. How do you distinguish between a geometric sequence and an arithmetic sequence?**

A geometric sequence involves a constant multiplicative factor called the common ratio between consecutive terms. In contrast, an arithmetic sequence involves a constant additive factor called the common difference between consecutive terms. You divide terms to find the ratio, and subtract terms to find the difference.

2. What is the primary strategy for solving word problems involving age, motion, or mixtures?

The primary strategy is to first define variables clearly for the unknown quantities. Then, translate the verbal descriptions into one or more algebraic equations based on the relationships given in the problem. Finally, solve the resulting equation or system of equations.

3. Explain how dimensional analysis helps in unit conversion problems.

Dimensional analysis helps by systematically multiplying the original measurement by conversion factors, which are ratios equal to one. By setting up these factors so that unwanted units cancel out and desired units remain, you can ensure the conversion is performed correctly and arrive at the correct final units.

4. When should you use a permutation versus a combination in a counting problem?

You should use a permutation when the order of selection or arrangement of items matters, such as arranging books on a shelf or awarding distinct prizes. You should use a combination when the order of selection does not matter, such as choosing a committee or selecting a group of items without regard to their sequence.

Chapter 5: SAT Math Review

1. How do you calculate the average of a set of numbers, and what if you need to find a missing number when the average is known?

To calculate the average, sum all the numbers in the set and divide by the total count of numbers. If the average is known along with some numbers, first calculate the total sum that corresponds to the given average and count. Then, subtract the sum of the known numbers from this total sum to find the missing number.

2. Explain the difference between simple interest and compound interest.

Simple interest is calculated solely on the principal amount for the entire duration of the investment or loan. Compound interest, however, is calculated on the principal amount plus any accumulated interest from prior periods. This means compound interest leads to faster growth of the investment or debt over time because interest itself earns interest.

3. What is the slope-intercept form of a linear equation, and how can it be used to understand a line's characteristics?

The slope-intercept form is $y = mx + b$, where 'm' represents the slope of the line and 'b' represents the y-intercept. From this form, you can immediately identify the line's steepness and direction (from 'm') and the point where it crosses the y-axis (from 'b'). This makes graphing and analyzing linear relationships straightforward.

4. Describe the angle relationships formed when two parallel lines are cut by a transversal.

When two parallel lines are intersected by a transversal, several angle pairs have specific relationships. Corresponding angles, alternate interior angles, and alternate exterior angles are all equal. Consecutive interior angles (or same-side interior angles) are supplementary, meaning their sum is 180 degrees.

5. What defines similar triangles, and how are proportions used with them?

Similar triangles are triangles that have the same shape but can differ in size. This means their corresponding angles are equal, and their corresponding side lengths are in proportion. Proportions

are used to set up ratios of corresponding sides, allowing you to solve for unknown side lengths by cross-multiplication.

Chapter 6: ACT Math Critical Concepts

1. How does multiplying or dividing an inequality by a negative number affect the inequality sign?

When multiplying or dividing an inequality by a negative number, the direction of the inequality sign must be reversed. For example, if you have $-2x > 6$, dividing by -2 changes it to $x < -3$. Failing to reverse the sign is a common error.

2. Explain the difference between a permutation and a combination.

A permutation is an arrangement of items where the order of selection matters. A combination is a selection of items where the order does not matter. For example, selecting a president and vice-president is a permutation, while selecting two members for a committee is a combination.

3. What is the purpose of the discriminant in a quadratic equation?

The discriminant, $b^2 - 4ac$, helps determine the nature and number of roots (solutions) of a quadratic equation. If it is positive, there are two distinct real roots. If it is zero, there is one real root. If it is negative, there are two complex conjugate roots.

4. Describe how to divide complex numbers.

To divide complex numbers, you multiply both the numerator and the denominator by the conjugate of the denominator. This process eliminates the imaginary unit from the denominator, resulting in a complex number in the standard $a + bi$ form. The conjugate of $c + di$ is $c - di$.

5. What are the formulas for the area and circumference of a circle?

The area of a circle is calculated using the formula $A = \pi r^2$, where r is the radius. The circumference of a circle is calculated using $C = 2\pi r$ or $C = \pi d$, where d is the diameter. These formulas are fundamental for solving circle-related geometry problems.

Chapter 7: Foundations Refresher

1. What is the crucial rule to remember when solving inequalities that differs from solving equations?

When solving an inequality, you must reverse the direction of the inequality sign if you multiply or divide both sides by a negative number. This is the only procedural difference from solving standard equations.

2. How can you quickly determine if two lines are parallel or perpendicular based on their slopes?

Parallel lines have identical slopes. Perpendicular lines have slopes that, when multiplied together, result in -1 . For example, slopes of $2/3$ and $-3/2$ indicate perpendicular lines.

3. Explain the 'power move' for solving average problems on tests.

The power move is to use the average formula backward: $\text{Total} = \text{Average} \times \text{Count}$. This allows you to

find the sum of values needed to achieve a certain average, which is crucial for problems involving adding new values to a set.

4. List two common Pythagorean triples and two special right triangle side ratios.

Common Pythagorean triples include 3-4-5 and 5-12-13, as well as their multiples. The side ratios for special right triangles are $1 : 1 : \sqrt{2}$ for a 45-45-90 triangle and $1 : \sqrt{3} : 2$ for a 30-60-90 triangle.

5. Why is it important to use official practice materials for SAT and ACT preparation?

Official practice materials accurately reflect the format, question types, and content of the actual exams. Using them ensures your preparation is relevant and helps you become familiar with the test environment, reducing surprises on test day.

Chapter 8: Final Assessment: Math Foundations

1. What is the primary purpose of the Final Assessment in this course?

The primary purpose is to serve as a diagnostic tool. It evaluates your mastery of the core math foundations covered in the course, highlighting both your strengths and areas that require further review before moving to advanced test strategies.

2. How should you prepare for and approach this assessment?

You should prepare by reviewing all previous modules and lessons. Approach the assessment like a real test: find a quiet place, read each question carefully, show your work, and manage your time effectively. This practice builds good test-taking habits.

3. What are some common mistakes students make on math assessments?

Common mistakes include careless arithmetic errors, misinterpreting word problems, forgetting key formulas or properties, and rushing through questions. Paying close attention to details and double-checking work can help avoid these pitfalls.

4. After completing the assessment, what is the most important step to take?

The most important step is to review all your answers, especially the incorrect ones. Analyze why you made each mistake, identify patterns in your errors, and pinpoint specific topics that need more practice. This analysis guides your future study.

5. Why is it important to rebuild math foundations before an SAT or ACT strategy course?

Rebuilding math foundations ensures you have a strong understanding of the underlying concepts tested on the SAT and ACT. Without this foundational knowledge, advanced test strategies would be less effective, as you would still struggle with the core mathematical operations and problem types.

CLOSING

Where to Go Next

After completing "SAT & ACT Math Foundations," you will have a strong mathematical base. Consolidate your skills by regularly practicing problems from various sources. Revisit any challenging topics in this guide or the course videos.

Your next natural step is to move into specific SAT or ACT test strategy courses. This foundational course has equipped you with the math skills needed to understand and apply advanced test-taking techniques. You can also begin taking full-length practice tests to apply your knowledge in a timed environment.

Continue to review your math fundamentals even as you move to strategy. Consistent practice and review will ensure your skills remain sharp for test day.



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