

OFFICIAL COURSE STUDY GUIDE

Yale: Fundamentals of Physics

Professor Shankar's acclaimed Yale physics course.

SCIENCE & MATH · ADVANCED · 3 MODULES · 28 HR 45 MIN

Course by YaleCourses · Curated and published by MegaCourse

MEGACOURSE.APP

Yale: Fundamentals of Physics: The Complete Study Guide

2026 Edition

Published by MegaCourse · megacourse.app

Learn Anything. Achieve Everything.

MegaCourse curates the world's best free courses into structured learning paths with study guides, practice material, and certificates. This guide accompanies the MegaCourse course "Yale: Fundamentals of Physics", taught by YaleCourses. It is designed to be read alongside the video lessons and to serve as a standalone reference for review.

Study guide text copyright © 2026 MegaCourse. Course video content remains the property of its creators. You may print this guide for personal study. Redistribution or resale is not permitted.

Enroll, track progress, and earn your certificate at
megacourse.app/courses/yale-fundamentals-of-physics

Contents

<i>About This Course</i>	<i>4</i>
CHAPTER 1 Newtonian Mechanics and Vectors	6
CHAPTER 2 Rotational Motion and Special Relativity	13
CHAPTER 3 Oscillations, Waves, Fluids, and Thermodynamics	18
<i>Glossary</i>	<i>24</i>
<i>Answer Key</i>	<i>26</i>
<i>Where to Go Next</i>	<i>29</i>

INTRODUCTION

About This Course

Welcome to the official study guide for Yale: Fundamentals of Physics. This course offers a rigorous introduction to the core principles of classical mechanics. You will explore the foundational concepts that describe how the universe operates at an everyday scale.

Physics is fundamental to understanding the world around us. From the motion of planets to the design of bridges, its principles govern everything. A strong grasp of physics develops critical thinking and problem-solving skills applicable across many disciplines.

Professor Shankar's Yale lectures are renowned for their clarity and depth. This MegaCourse learning path organizes his acclaimed instruction into a structured curriculum. We provide clear learning outcomes, track your progress, and offer this study guide to reinforce your understanding. You will build a solid university-level physics intuition.

The course is divided into 3 modules and 24 lessons, totaling nearly 29 hours of instruction. This structure helps you manage your learning effectively. Each lesson builds upon previous concepts, ensuring a coherent and comprehensive educational experience.

Course Facts

FORMAT	3 modules · 24 lessons
VIDEO INSTRUCTION	About 28 hr 45 min
LEVEL	Advanced
CATEGORY	Science & Math
INSTRUCTOR	YaleCourses
CERTIFICATE	Included with course completion on MegaCourse
COURSE HOME	megacourse.app/courses/yale-fundamentals-of-physics

What You Will Learn

- Understand Newtonian mechanics thoroughly.
- Solve complex problems in classical mechanics.
- Grasp the concepts of energy, momentum, and waves.
- Develop a strong university-level physics intuition.
- Analyze physical systems using mathematical models.

- Apply conservation laws to various physical scenarios.

Who This Course Is For

This course is designed for serious students seeking a comprehensive and challenging introduction to university-level physics. It is ideal for individuals preparing for higher education in science, engineering, or mathematics. It also serves those who wish to deepen their existing scientific knowledge.

Before beginning, you should possess a solid foundation in high school physics. Crucially, you must also be proficient in calculus. This includes differentiation, integration, and vector calculus. These mathematical tools are essential for understanding the derivations and solving the problems presented.

Before You Start

- Possess a solid understanding of high school physics.
- Be proficient in differential and integral calculus.
- Have a working knowledge of vector algebra.

How to Use This Guide

To get the most from this course, engage actively with both the video lectures and this study guide. Watch each video lesson carefully, taking notes as you go. Afterwards, review the corresponding sections in this guide. The guide provides summaries, additional explanations, and practice problems that complement the video material.

Do not rush through the material. Physics requires careful thought and repeated practice. Use the review questions at the end of each lesson to test your comprehension. Consult the glossary for definitions of key terms. Reworking problems and explaining concepts in your own words will solidify your understanding.

Consider forming a study group to discuss challenging concepts. Teaching a topic to someone else is an excellent way to identify gaps in your own knowledge. Attempt all practice problems before looking at solutions. This active engagement is vital for building true mastery.

CHAPTER 1

Newtonian Mechanics and Vectors

Module 1 introduces the foundational principles of classical mechanics. We begin with the mathematical language of physics, vectors, which are essential for describing motion and forces in multiple dimensions. The core of this module lies in Newton's three laws of motion. These laws provide the framework for understanding how forces cause objects to accelerate.

We then explore the concepts of work and energy, including the powerful principle of conservation of energy, which offers an alternative and often simpler approach to solving mechanics problems. Finally, we extend these ideas to systems of multiple particles and works through the dynamics of gravitational interactions, culminating in Kepler's laws of planetary motion. Mastering these concepts is crucial for building a solid foundation in university-level physics.

Foundations of Newtonian Mechanics and Vectors

Covers: 1. Course Introduction and Newtonian Mechanics · 2. Vectors in Multiple Dimensions

Newtonian mechanics describes the motion of objects under the influence of forces. It is built upon fundamental quantities: mass, length, and time. These are the building blocks for all other physical quantities in this domain. Understanding how these quantities relate and are measured is the first step in solving physics problems.

Many physical quantities require both a magnitude and a direction for a complete description. These are called vectors. Examples include displacement, velocity, acceleration, and force. Quantities with only magnitude, such as mass, time, temperature, and speed, are scalars. We represent vectors graphically as arrows, where the length indicates magnitude and the arrow points in the direction. We can also represent vectors using components along coordinate axes, typically x, y, and z. For example, a vector $\mathbf{A}$ in two dimensions can be written as $\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x and y axes, respectively.

Vector addition and subtraction follow specific rules. Graphically, we add vectors by placing them head-to-tail. The resultant vector goes from the tail of the first to the head of the last. Using components, vector addition is straightforward: sum the corresponding components. For instance, if $\mathbf{A} = (A_x, A_y)$ and $\mathbf{B} = (B_x, B_y)$, then $\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y)$. Subtraction is similar: $\mathbf{A} - \mathbf{B} = (A_x - B_x, A_y - B_y)$.

Two important ways to multiply vectors are the dot product and the cross product. The dot product (or scalar product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ yields a scalar quantity. It is defined as $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos(\theta)$, where θ is the angle between the vectors. In component form, $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$. The dot product is useful for finding the work done by a force or the angle between two vectors.

The cross product (or vector product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ yields a new vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$. Its magnitude is $|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}| |\mathbf{B}| \sin(\theta)$. The direction of the resulting vector is determined by the right-hand rule. If you curl the fingers of your right hand from $\mathbf{A}$ to $\mathbf{B}$, your thumb points in the

direction of $\mathbf{A} \times \mathbf{B}$. In component form, for 3D vectors: $\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \mathbf{i} + (A_z B_x - A_x B_z) \mathbf{j} + (A_x B_y - A_y B_x) \mathbf{k}$. The cross product is used to calculate torque and magnetic forces.

Practice Example: Vector Operations

Given vectors $\mathbf{A} = (3, 4)$ and $\mathbf{B} = (2, -1)$:

1. Find $\mathbf{A} + \mathbf{B}$.
2. Calculate $\mathbf{A} \cdot \mathbf{B}$.
3. Calculate the magnitude of $\mathbf{A}$.

1. $\mathbf{A} + \mathbf{B} = (3+2, 4+(-1)) = (5, 3)$
2. $\mathbf{A} \cdot \mathbf{B} = (3)(2) + (4)(-1) = 6 - 4 = 2$
3. $|\mathbf{A}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Common mistakes include confusing scalar and vector quantities, incorrectly resolving vectors into components, and misapplying the right-hand rule for the cross product. Always visualize the vectors and their components to avoid errors. Practice drawing vectors and performing operations graphically before relying solely on component calculations.

Newton's Laws of Motion and Force Applications

Covers: 3. Newton's Laws of Motion · 4. Newton's Laws (cont.) and Inclined Planes

Newton's three laws of motion form the bedrock of classical mechanics. His First Law, the Law of Inertia, states that an object at rest stays at rest, and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force. This law defines an inertial reference frame, where objects maintain constant velocity in the absence of net forces.

Newton's Second Law is quantitative: The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. This is expressed by the equation $\mathbf{F}_{\text{net}} = m\mathbf{a}$, where $\mathbf{F}_{\text{net}}$ is the vector sum of all forces, m is the mass, and $\mathbf{a}$ is the acceleration. The unit of force is the Newton (N), defined as $1 \text{ kg} \cdot \text{m/s}^2$. This law is central to solving dynamics problems, as it relates forces to motion. Remember that $\mathbf{F}_{\text{net}}$ and $\mathbf{a}$ are always in the same direction.

Newton's Third Law states that for every action, there is an equal and opposite reaction. If object A exerts a force on object B, then object B simultaneously exerts an equal and opposite force on object A. These action-reaction pairs always act on *different* objects. For example, when you push a wall, the wall pushes back on you with an equal force. A common mistake is to confuse action-reaction pairs with forces acting on the same object that happen to be equal and opposite, such as normal force and weight (unless the object is in freefall).

To apply Newton's Laws, we use free-body diagrams. A free-body diagram isolates a single object and shows all the forces acting *on* that object, represented as vectors originating from the object's center. Steps for drawing a free-body diagram:

1. Identify the object of interest.

2. Draw the object as a point or simple shape.
3. Identify all forces acting on the object.
4. Draw each force as an arrow, indicating its direction and point of application.
5. Choose a coordinate system that simplifies the problem, often aligning one axis with the acceleration.

Common forces include:

- **Weight (gravitational force):** $F_g = mg$, always directed downwards.
- **Normal force:** F_N , exerted by a surface perpendicular to the surface, preventing penetration.
- **Tension:** T , exerted by a rope or string, always pulling along the length of the rope.
- **Friction:** f , opposes relative motion between surfaces. Static friction ($f_s \leq \mu_s F_N$) prevents motion, kinetic friction ($f_k = \mu_k F_N$) opposes existing motion.

Example: Block on an Inclined Plane

A block of mass m rests on an inclined plane at an angle θ to the horizontal. The coefficient of kinetic friction is μ_k . Find the acceleration if the block slides down.

1. **Draw a free-body diagram:** Forces are weight (mg) vertically down, normal force (F_N) perpendicular to the plane, and kinetic friction (f_k) up the plane.
2. **Choose coordinate system:** Align x-axis parallel to the plane (downwards positive), y-axis perpendicular to the plane.
3. **Resolve forces:** Weight mg resolves into $mg \sin(\theta)$ (parallel to plane) and $mg \cos(\theta)$ (perpendicular to plane).
4. **Apply Newton's Second Law:**
 - In y-direction (no acceleration): $\sum F_y = F_N - mg \cos(\theta) = 0 \Rightarrow F_N = mg \cos(\theta)$.
 - In x-direction: $\sum F_x = mg \sin(\theta) - f_k = ma$.
5. **Substitute friction:** $f_k = \mu_k F_N = \mu_k mg \cos(\theta)$.
6. **Solve for acceleration:** $mg \sin(\theta) - \mu_k mg \cos(\theta) = ma$.

$$a = g (\sin(\theta) - \mu_k \cos(\theta))$$

This result shows the acceleration depends on gravity, the incline angle, and the friction coefficient. If μ_k is large enough, a might be zero or negative, meaning the block would not slide down or would accelerate up if pushed. Remember to always draw clear diagrams and resolve forces carefully to avoid sign errors.

Work, Energy, and Conservation

Covers: 5. Work-Energy Theorem and Law of Conservation of Energy · 6. Law of Conservation of Energy in Higher Dimensions

Work is a scalar quantity defined as the product of the force component in the direction of displacement and the magnitude of the displacement. For a constant force $\mathbf{F}$ causing a displacement $\mathbf{d}$, the work $W = \mathbf{F} \cdot$

$\mathbf{d} = |\mathbf{F}| |\mathbf{d}| \cos(\theta)$, where θ is the angle between the force and displacement vectors. The unit of work is the Joule (J), which is 1 N·m. If the force is perpendicular to the displacement, no work is done. If the force acts opposite to the displacement, the work done is negative.

Kinetic energy (KE) is the energy an object possesses due to its motion. It is defined as $KE = 0.5 * m * v^2$, where m is the mass and v is the speed. The Work-Energy Theorem states that the net work done on an object equals the change in its kinetic energy: $W_{\text{net}} = \Delta KE = KE_{\text{final}} - KE_{\text{initial}}$. This theorem provides a powerful alternative to Newton's laws for problems involving changes in speed and displacement, especially when forces are complex or vary.

Potential energy (PE) is stored energy due to an object's position or configuration. Two common types are gravitational potential energy and spring potential energy. Gravitational potential energy (PE_g) near the Earth's surface is $PE_g = mgh$, where m is mass, g is the acceleration due to gravity, and h is the height above a chosen reference level. Spring potential energy (PE_s) stored in an ideal spring is $PE_s = 0.5 * k * x^2$, where k is the spring constant and x is the displacement from the spring's equilibrium position.

Forces can be classified as conservative or non-conservative. A conservative force, like gravity or the spring force, does work that is independent of the path taken between two points. For these forces, we can define a potential energy. A non-conservative force, like friction or air resistance, does work that depends on the path taken. Mechanical energy (E) is the sum of kinetic and potential energies: $E = KE + PE$. The Law of Conservation of Mechanical Energy states that if only conservative forces do work, the total mechanical energy of a system remains constant: $E_{\text{initial}} = E_{\text{final}}$. If non-conservative forces do work, then the change in mechanical energy equals the work done by the non-conservative forces: $W_{\text{nc}} = \Delta E = E_{\text{final}} - E_{\text{initial}}$.

Power is the rate at which work is done or energy is transferred. It is defined as $P = W/t$. It can also be expressed as $P = \mathbf{F} \cdot \mathbf{v}$ for a constant force $\mathbf{F}$ and velocity $\mathbf{v}$. The unit of power is the Watt (W), which is 1 Joule/second.

Example: Roller Coaster Energy Conservation

A roller coaster car of mass 200 kg starts from rest at the top of a hill 50 m high. What is its speed at the bottom of the hill, assuming no friction?

- Identify initial and final states:** Initial (top of hill): $v_i = 0$, $h_i = 50$ m. Final (bottom of hill): $h_f = 0$, $v_f = ?$.
- Apply Conservation of Mechanical Energy:** Since no friction (a non-conservative force) is present, mechanical energy is conserved. $KE_i + PE_i = KE_f + PE_f$
 $0.5 * m * v_i^2 + mgh_i = 0.5 * m * v_f^2 + mgh_f$
- Substitute values:** We can cancel 'm' from all terms. $0 + g * h_i = 0.5 * v_f^2 + 0$
 $g * h_i = 0.5 * v_f^2$
- Solve for v_f :**

$$v_f = \sqrt{2 * g * h_i}$$

$$v_f = \sqrt{2 * 9.8 \text{ m/s}^2 * 50 \text{ m}}$$

$$v_f = \sqrt{980 \text{ m}^2/\text{s}^2}$$

$$v_f \approx 31.3 \text{ m/s}$$

Common mistakes include forgetting to account for work done by non-conservative forces, choosing inconsistent reference points for potential energy, or mixing up kinetic and potential energy terms. Always define your system and identify all forces acting within it or on it.

Systems of Particles and Gravitation

Covers: 7. Kepler's Laws · 8. Dynamics of Multiple-Body System and Law of

When dealing with multiple objects, it is often useful to consider the system's center of mass. The center of mass is the average position of all the mass in the system. For a system of discrete particles, the position of the center of mass ($\mathbf{R}_{CM}$) is given by $\mathbf{R}_{CM} = (\sum m_i \mathbf{r}_i) / (\sum m_i)$. The motion of the center of mass of a system is governed by the net external force acting on the system: $\mathbf{F}_{net,ext} = M\mathbf{a}_{CM}$, where M is the total mass of the system. This means the center of mass moves as if all the system's mass were concentrated there and all external forces acted upon it.

Linear momentum ($\mathbf{p}$) is a vector quantity defined as the product of an object's mass and its velocity: $\mathbf{p} = m\mathbf{v}$. The impulse ($\mathbf{J}$) applied to an object is the change in its momentum: $\mathbf{J} = \Delta \mathbf{p}$. Impulse can also be calculated as the average force multiplied by the time interval over which it acts: $\mathbf{J} = \mathbf{F}_{avg} * \Delta t$. The Law of Conservation of Linear Momentum states that if the net external force on a system is zero, then the total linear momentum of the system remains constant. This principle is particularly useful for analyzing collisions.

Collisions are interactions between objects that involve large forces over short time intervals. We classify collisions based on whether kinetic energy is conserved. In an elastic collision, both momentum and kinetic energy are conserved. In an inelastic collision, momentum is conserved, but kinetic energy is not; some kinetic energy is converted into other forms, like heat or sound. A perfectly inelastic collision is an extreme case of an inelastic collision where the colliding objects stick together after impact.

Newton's Law of Universal Gravitation describes the attractive force between any two objects with mass. The magnitude of this force is $F = G * (m_1 * m_2) / r^2$, where G is the universal gravitational constant (approximately $6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$), m_1 and m_2 are the masses of the objects, and r is the distance between their centers. This force is always attractive and acts along the line connecting the two objects. Gravitational potential energy for two masses is $U = -G * (m_1 * m_2) / r$, where the negative sign indicates an attractive force and the potential energy is zero at infinite separation.

Kepler's Laws describe planetary motion around a star:

1. **Law of Orbits:** Planets orbit the Sun in ellipses, with the Sun at one focus.
2. **Law of Areas:** A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time. This implies that planets move faster when they are closer to the Sun.
3. **Law of Periods:** The square of the orbital period (T) of a planet is directly proportional to the cube of the semi-major axis (a) of its orbit. For circular orbits, this simplifies to T^2 proportional to r^3 . This relationship can be derived from Newton's Law of Universal Gravitation and Newton's Second Law.

Example: Perfectly Inelastic Collision

A 2 kg block moving at 5 m/s collides with and sticks to a stationary 3 kg block. What is their final velocity?

1. **Identify system:** The two blocks constitute the system.
2. **Check for external forces:** Assuming the collision happens on a frictionless surface, the net external force is zero, so momentum is conserved.
3. **Apply Conservation of Momentum:** $m_1v_{1_initial} + m_2v_{2_initial} = (m_1 + m_2)v_{final}$

$$\begin{aligned}(2 \text{ kg})(5 \text{ m/s}) + (3 \text{ kg})(0 \text{ m/s}) &= (2 \text{ kg} + 3 \text{ kg}) * v_{final} \\ 10 \text{ kg}\cdot\text{m/s} &= 5 \text{ kg} * v_{final} \\ v_{final} &= 10 \text{ kg}\cdot\text{m/s} / 5 \text{ kg} = 2 \text{ m/s}\end{aligned}$$

The final velocity of the combined mass is 2 m/s. Common errors include forgetting that momentum is a vector, incorrectly applying conservation laws when external forces are present, or misinterpreting the conditions for elastic versus inelastic collisions. Always consider the system boundaries and external forces.

KEY TAKEAWAYS

- Vectors are essential for describing quantities with both magnitude and direction, and their operations like addition, dot product, and cross product are fundamental.
- Newton's three laws of motion provide the core framework for understanding how forces cause objects to accelerate, requiring careful application with free-body diagrams.
- Work is the transfer of energy, and the Work-Energy Theorem relates net work to changes in kinetic energy.
- The Law of Conservation of Mechanical Energy states that total mechanical energy remains constant if only conservative forces do work.
- Momentum is conserved in a system when no net external forces act upon it, a principle crucial for analyzing collisions.
- Newton's Law of Universal Gravitation describes the attractive force between any two masses, forming the basis for understanding orbital mechanics.
- Kepler's Laws describe the elliptical paths, varying speeds, and period-radius relationships of planetary orbits.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. Explain the difference between a scalar and a vector quantity, providing two examples of each.
2. What are Newton's three laws of motion? Briefly describe each.

3. How does the Work-Energy Theorem simplify problem-solving compared to using Newton's Second Law for certain situations?
4. Under what conditions is the total mechanical energy of a system conserved? What happens if these conditions are not met?
5. State the Law of Conservation of Linear Momentum and explain its application in analyzing collisions.

CHAPTER 2

Rotational Motion and Special Relativity

Chapter 2 introduces two fundamental areas of physics: rotational motion and special relativity. First, you will build upon your understanding of linear dynamics to explore how objects rotate. This includes concepts such as moment of inertia, angular momentum, and torque. You will learn to analyze the motion of rigid bodies and solve problems involving rotational equilibrium and dynamics.

Next, the chapter shifts to the revolutionary ideas of special relativity. This section challenges classical notions of space and time. You will learn about Einstein's postulates, the consequences of time dilation and length contraction, and the mathematical framework of Lorentz transformations. The chapter concludes by introducing four-vectors and their application to relativistic momentum and energy, along with a review of essential mathematical tools like the Taylor series.

Rotational Dynamics and Torque

Covers: 9. Rotations, Part I: Dynamics of Rigid Bodies · 10. Rotations, Part II: Parallel Axis Theorem · 11. Torque

Understanding rotational motion is key to describing how objects spin, roll, or pivot. Just as linear motion involves mass, force, and acceleration, rotational motion involves analogous quantities. The rotational equivalent of mass is the **moment of inertia** (I). This quantity depends on an object's mass and how that mass is distributed relative to the axis of rotation. A larger moment of inertia means an object is harder to start or stop rotating. For a point mass m at a distance r from the axis, $I = mr^2$. For extended objects, calculating I often involves integration.

Rotational kinetic energy is given by $K_{\text{rot}} = 0.5 * I * \omega^2$, where ω is the angular velocity. The angular momentum (L) of a rigid body is $L = I * \omega$. Like linear momentum, angular momentum is conserved in a closed system where no external torques act. This principle explains why a spinning ice skater speeds up when she pulls her arms in, decreasing her moment of inertia.

Calculating Moment of Inertia

For many common shapes, moments of inertia are known. For example, a solid cylinder rotating about its central axis has $I = 0.5 * M * R^2$. A thin rod rotating about its center has $I = (1/12) * M * L^2$. When the rotation axis is not through the center of mass, the **Parallel Axis Theorem** is indispensable. This theorem states $I = I_{\text{cm}} + M * d^2$, where I_{cm} is the moment of inertia about an axis through the center of mass, M is the total mass, and d is the perpendicular distance between the two parallel axes.

Let's consider a thin rod of mass M and length L . Its moment of inertia about an axis through its center is $I_{\text{cm}} = (1/12) * M * L^2$. If we want the moment of inertia about an axis at one end of the rod, the distance d is $L/2$. Using the Parallel Axis Theorem:

$$\begin{aligned}
 I_{\text{end}} &= I_{\text{cm}} + M * d^2 \\
 I_{\text{end}} &= (1/12) * M * L^2 + M * (L/2)^2 \\
 I_{\text{end}} &= (1/12) * M * L^2 + (1/4) * M * L^2 \\
 I_{\text{end}} &= (1/12) * M * L^2 + (3/12) * M * L^2 \\
 I_{\text{end}} &= (4/12) * M * L^2 \\
 I_{\text{end}} &= (1/3) * M * L^2
 \end{aligned}$$

This calculation shows how the theorem simplifies finding I for different axes. A common mistake is forgetting that 'd' must be the perpendicular distance between the parallel axes.

Torque (τ) is the rotational equivalent of force. It is what causes an object to angularly accelerate. Torque depends on the applied force, the distance from the pivot point to where the force is applied (the lever arm), and the angle between the force vector and the lever arm. Mathematically, $\tau = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is the position vector from the pivot to the point of force application and $\mathbf{F}$ is the force vector. The magnitude of torque is $\tau = rF\sin(\theta)$, where θ is the angle between $\mathbf{r}$ and $\mathbf{F}$. The net torque acting on a rigid body is related to its angular acceleration (α) by $\tau_{\text{net}} = I * \alpha$, which is the rotational analogue of Newton's second law, $F_{\text{net}} = ma$.

For an object to be in **static equilibrium**, two conditions must be met: the net force acting on it must be zero ($\Sigma \mathbf{F} = 0$), and the net torque acting on it must also be zero ($\Sigma \tau = 0$). When solving problems, choose a pivot point strategically to simplify torque calculations, often where an unknown force acts. Practice identifying the lever arm and the angle for various force applications. For example, a force applied directly towards or away from the pivot creates zero torque.

Foundations of Special Relativity: Time, Length, and Lorentz Transformations

Covers: 12. Introduction to Relativity · 13. Lorentz Transformation

Special relativity, developed by Albert Einstein, fundamentally changed our understanding of space and time. It is built upon two core postulates: first, the laws of physics are the same for all observers in uniform motion (inertial frames of reference). Second, the speed of light in a vacuum (c) is the same for all inertial observers, regardless of the motion of the light source. These postulates lead to profound and often counter-intuitive consequences.

One such consequence is the **relativity of simultaneity**. Events that appear simultaneous to one observer may not be simultaneous to another observer moving relative to the first. This means there is no absolute 'now' that applies universally. Another key effect is **time dilation**. A clock moving relative to an observer will appear to run slower than an identical clock at rest relative to the observer. The relationship is given by $t = \gamma * t_0$, where t_0 is the proper time (time measured by a clock at rest in its own frame), t is the dilated time measured by the moving observer, and γ is the Lorentz factor. The Lorentz factor is defined as $\gamma = 1 / \sqrt{1 - v^2/c^2}$, where v is the relative speed between the frames and c is the speed of light. Since v is always less than c , γ is always greater than or equal to 1.

For example, if a spaceship travels at $0.8c$ (80% of the speed of light), its Lorentz factor is:

$$\begin{aligned}
 \gamma &= 1 / \sqrt{1 - (0.8c)^2/c^2} \\
 \gamma &= 1 / \sqrt{1 - 0.64c^2/c^2}
 \end{aligned}$$

$$\begin{aligned}\gamma &= 1 / \sqrt{1 - 0.64} \\ \gamma &= 1 / \sqrt{0.36} \\ \gamma &= 1 / 0.6 \\ \gamma &\approx 1.67\end{aligned}$$

This means that for every 1 second that passes on a clock aboard the spaceship ($t_0 = 1$ s), an observer on Earth would measure approximately 1.67 seconds passing ($t = 1.67 * 1 \text{ s} = 1.67 \text{ s}$).

Similarly, **length contraction** predicts that the length of an object moving relative to an observer will appear shorter in the direction of motion compared to its length when measured at rest. The formula is $L = L_0 / \gamma$, where L_0 is the proper length (length measured in the object's rest frame) and L is the contracted length. For the spaceship moving at $0.8c$, if it has a proper length of 100 meters, an Earth observer would measure its length as $L = 100 \text{ m} / 1.67 \approx 60$ meters.

These effects are not optical illusions; they are real changes in the fabric of spacetime. A common mistake is applying length contraction or time dilation to the wrong frame of reference. Remember, proper time and proper length are measured in the object's *rest frame*.

The **Lorentz transformation** equations provide a mathematical framework to relate the coordinates (x, y, z, t) of an event in one inertial frame to the coordinates (x', y', z', t') of the same event in another inertial frame moving at a constant velocity v along the x -axis. The transformations are:

$$\begin{aligned}x' &= \gamma(x - vt) \\ t' &= \gamma(t - vx/c^2) \\ y' &= y \\ z' &= z\end{aligned}$$

These equations replace the Galilean transformations of classical physics. They show how space and time coordinates are intertwined. For example, the time coordinate t' in the moving frame depends not only on t but also on x , demonstrating the relativity of simultaneity. Practice using these equations to transform events between frames and to derive relativistic velocity addition formulas. For small velocities ($v \ll c$), the Lorentz factor γ approaches 1, and the Lorentz transformations reduce to the classical Galilean transformations.

Four-Vectors in Relativity and Essential Mathematical Tools

Covers: 14. Introduction to the Four-Vector · 15. Four-Vector in Relativity · 16. The Taylor Series and Other Mathematical Concepts

To fully appreciate the unified nature of spacetime in special relativity, physicists use **four-vectors**. A four-vector is a mathematical object with four components that transforms predictably under Lorentz transformations. This allows for a more elegant and covariant formulation of relativistic physics. The most fundamental is the **position four-vector**, which combines time and space coordinates: (ct, x, y, z) . Here, ct is used to give time the same units as space. The invariant quantity in spacetime is the **spacetime interval** (Δs^2), given by $(c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$. This interval is the same for all inertial observers, much like the distance between two points is invariant under rotation in Euclidean space. The proper time ($\Delta\tau$) is directly related to the invariant interval by $\Delta s^2 = (c\Delta\tau)^2$.

Another crucial four-vector is the **momentum four-vector**, also known as the energy-momentum four-vector. Its components are $(E/c, p_x, p_y, p_z)$, where E is the total relativistic energy and p_x, p_y, p_z are the components of relativistic momentum. The relativistic momentum of a particle is given by $\mathbf{p} = \gamma m \mathbf{v}$, where m is the rest mass and $\mathbf{v}$ is the velocity. The total relativistic energy is $E = \gamma mc^2$. This total energy includes both the kinetic energy and the rest energy ($E_0 = mc^2$). The famous equation $E = mc^2$ is a special case representing the energy of a particle at rest.

These quantities are related by the fundamental **energy-momentum relation**: $E^2 = (pc)^2 + (mc^2)^2$. This equation shows that energy and momentum are not independent but are components of a single four-vector. For a massless particle, like a photon, $m = 0$, so $E = pc$. The conservation of four-momentum is a powerful tool for analyzing collisions and decays in relativistic scenarios.

The Taylor Series and Its Applications

Physics often requires approximating complex functions, especially when dealing with limiting cases. The **Taylor series** is an essential mathematical tool for this purpose. It allows us to represent a function as an infinite sum of terms, calculated from the function's derivatives at a single point. The general form of a Taylor series for a function $f(x)$ about a point a is:

$$f(x) = f(a) + f'(a)(x-a)/1! + f''(a)(x-a)^2/2! + f'''(a)(x-a)^3/3! + \dots$$

A common application in physics is expanding functions around $x = 0$, which is called a Maclaurin series. For example, the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$ can be expanded using the generalized binomial theorem (a form of Taylor series) for low velocities ($v \ll c$):

$$\gamma \approx 1 + (1/2)(v^2/c^2) + (3/8)(v^2/c^2)^2 + \dots$$

When we expand the relativistic kinetic energy, $K = E - mc^2 = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$, and substitute the Taylor expansion for γ , we get:

$$\begin{aligned} K &\approx mc^2 [(1 + (1/2)(v^2/c^2) + \dots) - 1] \\ K &\approx mc^2 [(1/2)(v^2/c^2)] \\ K &\approx (1/2)mv^2 \end{aligned}$$

This shows that at low speeds, the relativistic kinetic energy correctly reduces to the classical kinetic energy, $0.5mv^2$. This demonstrates the consistency between classical and relativistic mechanics in their respective domains. Practice deriving these approximations using Taylor series for common functions. Understanding other mathematical concepts, such as vector calculus, differential equations, and linear algebra, also proves invaluable for advanced physics study.

KEY TAKEAWAYS

- Moment of inertia quantifies an object's resistance to rotational motion, depending on mass and its distribution relative to the axis of rotation.
- Torque is the rotational equivalent of force, causing angular acceleration, and is calculated as the cross product of the position vector and the force vector.
- The Parallel Axis Theorem simplifies moment of inertia calculations when the axis of rotation is not through the center of mass.
- Special relativity is based on the constancy of the speed of light and the principle of relativity, leading to time dilation and length contraction.
- Lorentz transformations provide the mathematical framework for converting space and time coordinates between different inertial reference frames.
- Four-vectors combine space and time or energy and momentum into single entities, allowing for a unified description of relativistic phenomena.
- The Taylor series is a powerful mathematical tool for approximating functions and showing how relativistic formulas reduce to classical ones at low speeds.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. Explain how the moment of inertia differs from mass, and provide an example of how it affects rotational motion.
2. What are the two conditions for static equilibrium, and why are both necessary?
3. Describe the two postulates of special relativity and their most direct consequences.
4. How do the Lorentz transformations differ from Galilean transformations, and when must they be used?
5. What is a four-vector in the context of special relativity, and what is its significance?
6. How can the Taylor series be used to connect relativistic physics with classical physics?

CHAPTER 3

Oscillations, Waves, Fluids, and Thermodynamics

This chapter introduces fundamental concepts in oscillations, waves, fluid mechanics, and thermodynamics. You will begin by exploring Simple Harmonic Motion, a foundational model for many periodic phenomena. The discussion then extends to the behavior of waves, examining their properties and interactions. Next, the principles governing fluids, both at rest and in motion, are presented. The chapter concludes with a comprehensive study of thermodynamics, covering energy transfer, the laws of thermodynamics, and the concept of entropy. Mastering these topics provides essential tools for understanding a wide range of physical systems.

Simple Harmonic Motion

Covers: 17. Simple Harmonic Motion (video, 1 hr 14 min) · 18. Simple Harmonic Motion (cont.) and Introduction to Waves (video, 1 hr 15 min)

Simple Harmonic Motion (SHM) describes a special type of oscillation where the restoring force is directly proportional to the displacement from equilibrium and acts in the opposite direction. This is often represented by Hooke's Law, $F = -kx$, where F is the restoring force, k is the spring constant, and x is the displacement. The negative sign indicates the force always pulls the object back to equilibrium.

Many physical systems approximate SHM. A mass attached to a spring oscillating on a frictionless surface is a classic example. A simple pendulum, for small angles of displacement, also exhibits SHM. For such systems, the motion is sinusoidal. The position of an object in SHM can be described by the equation $x(t) = A \cos(\omega t + \phi)$. Here, A is the amplitude, the maximum displacement from equilibrium. ω (omega) is the angular frequency, measured in radians per second. t is time, and ϕ (phi) is the phase constant, which depends on the initial conditions of the motion.

The angular frequency ω is a crucial parameter. For a mass m on a spring with constant k , $\omega = \sqrt{k/m}$. For a simple pendulum of length L in a gravitational field g (for small angles), $\omega = \sqrt{g/L}$. From angular frequency, we can find the period T , the time for one complete oscillation, using $T = 2\pi/\omega$. The frequency f , the number of oscillations per second, is given by $f = 1/T = \omega/(2\pi)$.

Energy is conserved in SHM. The total mechanical energy is the sum of kinetic energy (KE) and potential energy (PE). For a spring-mass system, $KE = \frac{1}{2}mv^2$ and $PE = \frac{1}{2}kx^2$. At maximum displacement ($x = A$, $v = 0$), all energy is potential: $E = \frac{1}{2}kA^2$. At equilibrium ($x = 0$, $v = v_{\max}$), all energy is kinetic: $E = \frac{1}{2}mv_{\max}^2$. Throughout the motion, $KE + PE$ remains constant. Understanding this energy transformation helps predict velocity at any position.

Example: Spring-Mass System

A 0.5 kg mass is attached to a spring with a spring constant of 50 N/m. It is stretched 0.1 m from equilibrium and released from rest.

1. Calculate the angular frequency ω .

$$\omega = \sqrt{k/m} = \sqrt{50 \text{ N/m} / 0.5 \text{ kg}} = \sqrt{100} = 10 \text{ rad/s}$$

2. Determine the period T and frequency f .

$$T = 2\pi/\omega = 2\pi/10 = \pi/5 \approx 0.628 \text{ s}$$

$$f = 1/T = 10/(2\pi) \approx 1.59 \text{ Hz}$$

3. Write the equation for its position $x(t)$. Since it's released from rest at $x = 0.1 \text{ m}$, the amplitude $A = 0.1 \text{ m}$. At $t = 0$, $x(0) = A$, so $\cos(\phi)$ must be 1, meaning $\phi = 0$.

$$x(t) = 0.1 \cos(10t) \text{ m}$$

Common mistakes include confusing angular frequency (ω) with regular frequency (f) and incorrectly determining the phase constant (ϕ) from initial conditions. Always check the units and ensure consistency.

Waves and Their Properties

Covers: 18. Simple Harmonic Motion (cont.) and Introduction to Waves (video, 1 hr 15 min) · 19. Waves (video, 1 hr 12 min)

Waves are disturbances that propagate through a medium, transferring energy without transferring matter. There are two primary types of mechanical waves: transverse and longitudinal. In a **transverse wave**, the particles of the medium oscillate perpendicular to the direction of wave propagation. Examples include waves on a string or electromagnetic waves. In a **longitudinal wave**, the particles oscillate parallel to the direction of wave propagation. Sound waves are a common example, consisting of compressions and rarefactions.

Key characteristics describe a wave: the **amplitude (A)** is the maximum displacement of a particle from its equilibrium position. The **wavelength (λ)** is the spatial period of the wave, the distance over which the wave's shape repeats. The **frequency (f)** is the number of complete oscillations a point on the wave makes per unit time. The **period (T)** is the time it takes for one complete oscillation, related to frequency by $T = 1/f$. The **wave speed (v)** is the speed at which the disturbance propagates through the medium. These quantities are related by the fundamental wave equation: $v = f\lambda$.

Waves carry energy. The energy transported by a wave is proportional to the square of its amplitude. This means a wave with twice the amplitude carries four times the energy. When two or more waves overlap, their displacements add algebraically. This is known as the **superposition principle**. If waves are in phase, their amplitudes add up, resulting in **constructive interference**. If they are out of phase, their amplitudes subtract, leading to **destructive interference**.

An important phenomenon is the formation of **standing waves**. These occur when two waves of the same frequency, wavelength, and amplitude travel in opposite directions and interfere. Standing waves appear stationary, with points of zero displacement called **nodes** and points of maximum displacement called **antinodes**. The conditions for standing waves depend on the boundary conditions of the medium.

For a string fixed at both ends, standing waves can only form if the length of the string is an integer multiple of half-wavelengths ($L = n\lambda/2$, where $n = 1, 2, 3, \dots$). The lowest frequency standing wave ($n=1$) is called the fundamental frequency or first harmonic. Higher frequencies are called harmonics or overtones.

Example: Wave on a String

A transverse wave on a string has a wavelength of 0.5 m and a frequency of 10 Hz. Calculate its wave speed.

Given: $\lambda = 0.5 \text{ m}$, $f = 10 \text{ Hz}$
Formula: $v = f\lambda$
Calculation: $v = (10 \text{ Hz}) * (0.5 \text{ m}) = 5 \text{ m/s}$

If the string is 1.25 m long and fixed at both ends, what is the wavelength of the fundamental (first harmonic) standing wave?

For a string fixed at both ends, the fundamental ($n=1$) has $L = \lambda/2$.
So, $\lambda = 2L = 2 * 1.25 \text{ m} = 2.5 \text{ m}$.

Remember that wave speed depends on the medium, while frequency depends on the source. Wavelength then adjusts according to $v = f\lambda$. A common mistake is to confuse the speed of the wave propagation with the speed of the particles oscillating within the medium.

Fluid Dynamics and Statics

Covers: 20. Fluid Dynamics and Statics and Bernoulli's Equation (video, 1 hr 13 min)

Fluids are substances that flow, encompassing liquids and gases. The study of fluids is divided into fluid statics (fluids at rest) and fluid dynamics (fluids in motion).

In **fluid statics**, we examine concepts like pressure and buoyancy. Pressure (P) is force per unit area, $P = F/A$, measured in Pascals (Pa) or N/m^2 . The density (ρ) of a fluid is its mass per unit volume, $\rho = m/V$. In a fluid at rest, pressure increases with depth. The pressure at a depth h below the surface of a fluid with density ρ is given by $P = P_0 + \rho gh$, where P_0 is the pressure at the surface. This principle explains why deep-sea divers experience immense pressure.

Pascal's Principle states that a pressure change applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container. This is the operating principle behind hydraulic systems, where a small force applied to a small piston can generate a large force on a larger piston.

Archimedes' Principle describes buoyancy. It states that an object partially or fully submerged in a fluid experiences an upward buoyant force equal to the weight of the fluid displaced by the object. The buoyant force (F_b) is calculated as $F_b = \rho_{\text{fluid}} * V_{\text{displaced}} * g$, where ρ_{fluid} is the density of the fluid, $V_{\text{displaced}}$ is the volume of fluid displaced, and g is the acceleration due to gravity. An object floats if its density is less than the fluid's density.

Fluid dynamics studies fluids in motion. For simplicity, we often consider an ideal fluid, which is incompressible (constant density), non-viscous (no internal friction), has steady flow (velocity at any point is constant over time), and is irrotational. Two key equations govern ideal fluid flow:

1. **The Continuity Equation:** This expresses the conservation of mass for an incompressible fluid. For a fluid flowing through a pipe, the product of the cross-sectional area (A) and the fluid speed (v) is constant: $A_1v_1 = A_2v_2$. This means fluid speeds up in narrower sections and slows down in wider sections.
2. **Bernoulli's Equation:** This is a statement of energy conservation for an ideal fluid. It relates pressure, speed, and height along a streamline: $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$. Here, P is the absolute pressure, ρ is the fluid density, v is the fluid speed, g is the acceleration due to gravity, and h is the height above a reference point. Bernoulli's equation shows that if fluid speed increases, its pressure must decrease, assuming constant height. This principle explains phenomena like the lift on an airplane wing and the Venturi effect.

Example: Bernoulli's Equation Application

Water flows through a horizontal pipe. At point 1, the pipe diameter is 4 cm, and the water speed is 2 m/s. At point 2, the diameter is 2 cm. What is the pressure difference between point 1 and point 2 if the pressure at point 1 is 150 kPa? (Density of water = 1000 kg/m³).

1. **Find the speed at point 2 using the Continuity Equation.** $A_1 = \pi(D_1/2)^2 = \pi(0.02 \text{ m})^2$ $A_2 = \pi(D_2/2)^2 = \pi(0.01 \text{ m})^2$ $A_1v_1 = A_2v_2 \Rightarrow v_2 = v_1 * (A_1/A_2) = v_1 * (D_1/D_2)^2$

$$v_2 = 2 \text{ m/s} * (4 \text{ cm} / 2 \text{ cm})^2 = 2 \text{ m/s} * (2)^2 = 8 \text{ m/s}$$

2. **Apply Bernoulli's Equation.** Since the pipe is horizontal, $h_1 = h_2$. $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$
 $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$$\begin{aligned} P_2 &= 150,000 \text{ Pa} + 0.5 * 1000 \text{ kg/m}^3 * ((2 \text{ m/s})^2 - (8 \text{ m/s})^2) \\ P_2 &= 150,000 \text{ Pa} + 500 * (4 - 64) \text{ Pa} \\ P_2 &= 150,000 \text{ Pa} + 500 * (-60) \text{ Pa} \\ P_2 &= 150,000 \text{ Pa} - 30,000 \text{ Pa} = 120,000 \text{ Pa} = 120 \text{ kPa} \end{aligned}$$

The pressure difference is $P_1 - P_2 = 150 \text{ kPa} - 120 \text{ kPa} = 30 \text{ kPa}$. The pressure decreases where the speed increases. A common error is neglecting to convert units to SI or incorrectly applying the continuity equation, especially with diameters versus radii.

Thermodynamics and Its Laws

Covers: 21. Thermodynamics (video, 1 hr 11 min) · 22. The Boltzmann Constant and First Law of Thermodynamics (video, 1 hr 15 min) · 23. The Second Law of Thermodynamics and Carnot's Engine (video, 1 hr 11 min) · 24. The Second Law of Thermodynamics (cont.) and Entropy (video, 1 hr 12 min)

Thermodynamics is the branch of physics concerned with heat and its relation to other forms of energy and work. It describes how thermal energy is converted into other forms of energy and how energy is

transferred between systems and their surroundings. Key concepts include temperature, heat, internal energy, and entropy.

Temperature is a measure of the average kinetic energy of the particles within a substance. Heat (Q) is the transfer of thermal energy due to a temperature difference. Internal energy (U) is the total energy contained within a system, including kinetic and potential energy of its molecules.

The **Zeroth Law of Thermodynamics** establishes the concept of temperature. It states that if two systems are each in thermal equilibrium with a third system, then they are in thermal equilibrium with each other. This allows for the definition of temperature as a property that determines whether systems are in thermal equilibrium.

The **First Law of Thermodynamics** is a statement of energy conservation. It states that the change in internal energy (ΔU) of a system equals the heat (Q) added to the system minus the work (W) done *by* the system: $\Delta U = Q - W$. If work is done *on* the system, the equation becomes $\Delta U = Q + W$. The sign convention for Q and W is critical. Heat added to the system is positive; heat leaving is negative. Work done by the system is positive; work done on the system is negative. For an ideal gas, the work done in a volume change is $W = \int P \, dV$. This work can be visualized as the area under a pressure-volume (PV) diagram.

An **ideal gas** follows the equation $PV = nRT$, where P is pressure, V is volume, n is the number of moles, R is the ideal gas constant ($8.314 \text{ J}/(\text{mol}\cdot\text{K})$), and T is the absolute temperature. Alternatively, $PV = NkT$, where N is the number of particles and k is the Boltzmann constant ($1.38 \times 10^{-23} \text{ J/K}$). The Boltzmann constant relates the average kinetic energy of particles to the absolute temperature.

The **Second Law of Thermodynamics** dictates the direction of natural processes. It can be stated in several ways: heat flows spontaneously from hotter objects to colder objects, never the reverse. Another statement is that it is impossible to construct a heat engine that operates in a cycle and produces no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work. This implies that no heat engine can be 100% efficient. The most efficient possible engine is the **Carnot engine**, operating in a reversible Carnot cycle. Its efficiency is given by $\eta = 1 - T_c/T_h$, where T_c is the absolute temperature of the cold reservoir and T_h is the absolute temperature of the hot reservoir.

The Second Law also introduces **entropy (S)**, a measure of the disorder or randomness of a system. The total entropy of an isolated system can only increase or remain constant; it never decreases. For a reversible process, the change in entropy is $\Delta S = Q/T$. For irreversible processes, $\Delta S > Q/T$. The universe as a whole is an isolated system, so its total entropy is always increasing.

Example: First Law of Thermodynamics

A gas in a cylinder is heated by adding 500 J of heat. The gas expands, doing 200 J of work on its surroundings. What is the change in the internal energy of the gas?

Given: $Q = +500 \text{ J}$ (heat added to the system)
 $W = +200 \text{ J}$ (work done by the system)
Formula: $\Delta U = Q - W$
Calculation: $\Delta U = 500 \text{ J} - 200 \text{ J} = 300 \text{ J}$

The internal energy of the gas increases by 300 J. If the gas had been compressed, and 200 J of work was done *on* the gas, W would be -200 J, leading to $\Delta U = 500 \text{ J} - (-200 \text{ J}) = 700 \text{ J}$. Pay close attention to the sign conventions for heat and work.

KEY TAKEAWAYS

- Simple Harmonic Motion describes oscillations where the restoring force is proportional to displacement, leading to sinusoidal motion and energy conservation.
- Waves transfer energy without transferring matter, characterized by amplitude, wavelength, frequency, and speed, and obey the principle of superposition.
- Fluid statics involves pressure and buoyancy, while fluid dynamics uses the continuity equation and Bernoulli's equation to describe fluid flow.
- The Zeroth Law defines temperature, and the First Law of Thermodynamics states that energy is conserved ($\Delta U = Q - W$).
- The Ideal Gas Law ($PV = nRT$ or $PV = NkT$) relates pressure, volume, temperature, and the amount of gas.
- The Second Law of Thermodynamics states that entropy of an isolated system always increases and limits the efficiency of heat engines.
- The Carnot engine provides the theoretical maximum efficiency for a heat engine, determined by the temperatures of its hot and cold reservoirs.

CHECK YOUR UNDERSTANDING

Answer these before moving on. Model answers are in the Answer Key at the back of this guide.

1. What are the conditions for an object to undergo Simple Harmonic Motion, and how is its period determined?
2. Explain the relationship between wave speed, frequency, and wavelength. What is the superposition principle?
3. How do the Continuity Equation and Bernoulli's Equation apply to fluid flow?
4. State the First Law of Thermodynamics and explain the meaning of each term.
5. What is the significance of the Second Law of Thermodynamics regarding heat engines and entropy?

REFERENCE

Glossary

Bernoulli's equation

A statement of energy conservation for an ideal fluid in motion, relating pressure, speed, and height along a streamline: $P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$.

Boltzmann constant (k)

A physical constant relating the average kinetic energy of particles in a gas to the absolute temperature of the gas ($k = 1.38 \times 10^{-23} \text{ J/K}$).

Conservation of mechanical energy

The principle that the total mechanical energy (kinetic plus potential) of a system remains constant if only conservative forces do work.

Entropy (s)

A thermodynamic property that measures the disorder or randomness of a system, tending to increase in isolated systems.

First law of thermodynamics

A statement of energy conservation: the change in internal energy of a system equals the heat added to the system minus the work done by the system ($\Delta U = Q - W$).

Four-vector

A mathematical object with four components that transforms predictably under Lorentz transformations, used to describe events in spacetime or relativistic momentum and energy.

Free-body diagram

A diagram that isolates an object and shows all the forces acting on it, represented as vectors.

Kepler's laws

Three laws describing the motion of planets around the Sun, including elliptical orbits and specific relationships between orbital period and radius.

Length contraction

The phenomenon in special relativity where the length of an object moving relative to an observer appears shorter in the direction of motion.

Lorentz factor (γ)

A factor that appears in the equations of special relativity, quantifying the relativistic effects of time dilation and length contraction, dependent on relative velocity.

Moment of inertia

A measure of an object's resistance to changes in its rotational motion, analogous to mass in linear motion.

Momentum

A vector quantity defined as the product of an object's mass and its velocity ($p = mv$).

Newton's second law

States that the net force acting on an object is equal to the product of its mass and acceleration ($F_{\text{net}} = ma$).

Parallel axis theorem

A theorem used to calculate the moment of inertia of a rigid body about any axis, given its moment of inertia about a parallel axis through its center of mass.

Second law of thermodynamics

States that the total entropy of an isolated system can only increase or remain constant, and heat flows spontaneously from hot to cold.

Simple harmonic motion (shm)

Oscillatory motion where the restoring force is directly proportional to the displacement from equilibrium and acts in the opposite direction.

Taylor series

A representation of a function as an infinite sum of terms, calculated from the values of the function's derivatives at a single point.

Time dilation

The phenomenon in special relativity where a moving clock appears to run slower than an identical clock at rest relative to an observer.

Torque

A rotational force that causes an object to rotate or change its rotational motion, calculated as the product of force and lever arm.

Vector

A physical quantity that has both magnitude and direction, such as displacement, velocity, or force.

Wavelength (λ)

The spatial period of a wave, the distance over which the wave's shape repeats.

Work-energy theorem

States that the net work done on an object equals the change in its kinetic energy ($W_{\text{net}} = \Delta KE$).

REFERENCE

Answer Key

Chapter 1: Newtonian Mechanics and Vectors

1. **Explain the difference between a scalar and a vector quantity, providing two examples of each.**

A scalar quantity has only magnitude, while a vector quantity has both magnitude and direction. Examples of scalars include mass and temperature. Examples of vectors include velocity and force.

2. **What are Newton's three laws of motion? Briefly describe each.**

Newton's First Law (Inertia) states an object maintains constant velocity unless acted upon by a net force. His Second Law ($F=ma$) quantifies the relationship between force, mass, and acceleration. His Third Law (Action-Reaction) states that for every action, there is an equal and opposite reaction force acting on different objects.

3. **How does the Work-Energy Theorem simplify problem-solving compared to using Newton's Second Law for certain situations?**

The Work-Energy Theorem relates the net work done to the change in kinetic energy, bypassing the need to directly calculate acceleration and time. It is particularly useful when forces vary or when only initial and final speeds and positions are known, as it deals with scalar quantities of energy.

4. **Under what conditions is the total mechanical energy of a system conserved? What happens if these conditions are not met?**

The total mechanical energy of a system is conserved if only conservative forces (like gravity or spring force) do work. If non-conservative forces (like friction or air resistance) do work, then mechanical energy is not conserved; the work done by these non-conservative forces equals the change in the system's mechanical energy.

5. **State the Law of Conservation of Linear Momentum and explain its application in analyzing collisions.**

The Law of Conservation of Linear Momentum states that if the net external force on a system is zero, the total linear momentum of the system remains constant. In collisions, this law allows us to relate the total momentum before the collision to the total momentum after, regardless of whether kinetic energy is conserved (elastic vs. inelastic collisions).

Chapter 2: Rotational Motion and Special Relativity

1. **Explain how the moment of inertia differs from mass, and provide an example of how it affects rotational motion.**

Moment of inertia is the rotational analogue of mass, but it also depends on how mass is distributed relative to the axis of rotation. A larger moment of inertia means greater resistance to angular acceleration. For example, a solid cylinder and a hollow cylinder of the same mass and radius will have

different moments of inertia, with the hollow cylinder having a larger I if rotated about its central axis, making it harder to spin up.

2. What are the two conditions for static equilibrium, and why are both necessary?

For static equilibrium, both the net force and the net torque acting on an object must be zero. The net force being zero ensures that the object does not undergo linear acceleration. The net torque being zero ensures that the object does not undergo angular acceleration. Both conditions are necessary to guarantee that the object remains completely at rest without translating or rotating.

3. Describe the two postulates of special relativity and their most direct consequences.

The two postulates are: the laws of physics are the same for all inertial observers, and the speed of light in a vacuum is constant for all inertial observers. Their direct consequences include time dilation, where moving clocks run slower, and length contraction, where moving objects appear shorter in their direction of motion, both relative to an observer.

4. How do the Lorentz transformations differ from Galilean transformations, and when must they be used?

Lorentz transformations account for the constancy of the speed of light and the relativity of simultaneity, intertwining space and time coordinates. Galilean transformations assume absolute time and independent space coordinates. Lorentz transformations must be used when dealing with relative velocities that are a significant fraction of the speed of light, where relativistic effects become noticeable.

5. What is a four-vector in the context of special relativity, and what is its significance?

A four-vector is a quantity with four components (one time-like, three space-like) that transforms predictably under Lorentz transformations, representing a unified entity in spacetime. Its significance lies in providing a covariant way to describe physical quantities like position, momentum, and energy, ensuring that physical laws maintain the same form in all inertial frames.

6. How can the Taylor series be used to connect relativistic physics with classical physics?

The Taylor series allows us to approximate relativistic equations for low velocities ($v \ll c$). By expanding the Lorentz factor or relativistic energy in a Taylor series, we can show that the higher-order terms become negligible, and the relativistic formulas reduce to their classical counterparts. This demonstrates that classical physics is a valid approximation of relativistic physics at everyday speeds.

Chapter 3: Oscillations, Waves, Fluids, and Thermodynamics

1. What are the conditions for an object to undergo Simple Harmonic Motion, and how is its period determined?

An object undergoes SHM if the net restoring force acting on it is directly proportional to its displacement from equilibrium and directed towards that equilibrium position. The period of SHM, the time for one complete oscillation, is determined by the system's properties, such as mass and spring constant for a spring-mass system, or length and gravity for a simple pendulum.

2. Explain the relationship between wave speed, frequency, and wavelength. What is the superposition principle?

Wave speed (v), frequency (f), and wavelength (λ) are related by the equation $v = f\lambda$. The wave speed is

determined by the medium, while the frequency is determined by the source. The superposition principle states that when two or more waves overlap, the resultant displacement at any point is the algebraic sum of the displacements due to individual waves.

3. How do the Continuity Equation and Bernoulli's Equation apply to fluid flow?

The Continuity Equation ($A_1v_1 = A_2v_2$) describes the conservation of mass for an incompressible fluid, showing that fluid speed increases in narrower sections of a pipe. Bernoulli's Equation ($P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$) describes the conservation of energy for an ideal fluid, relating pressure, speed, and height along a streamline.

4. State the First Law of Thermodynamics and explain the meaning of each term.

The First Law of Thermodynamics is $\Delta U = Q - W$. ΔU represents the change in the internal energy of the system. Q is the heat added to the system. W is the work done by the system. This law is a statement of energy conservation, indicating that energy cannot be created or destroyed, only transformed.

5. What is the significance of the Second Law of Thermodynamics regarding heat engines and entropy?

The Second Law of Thermodynamics places fundamental limits on the efficiency of heat engines, stating that no engine can be 100% efficient and that heat cannot spontaneously flow from a colder to a hotter body. It also introduces entropy, stating that the total entropy of an isolated system (like the universe) can only increase or remain constant, never decrease, reflecting the natural tendency towards greater disorder.

CLOSING

Where to Go Next

Upon completing Yale: Fundamentals of Physics, you will have a robust foundation in classical mechanics. To consolidate your skills, continue practicing problem-solving. Seek out new, challenging problems from textbooks or online resources. Revisit earlier lessons to reinforce any areas where you felt less confident.

Consider applying your knowledge to practical scenarios or personal projects. Explore how these principles manifest in engineering, astronomy, or sports. Engaging with real-world applications helps deepen your appreciation and understanding of physics.

The natural progression from this course involves studying electromagnetism, thermodynamics, and quantum mechanics. These subjects build directly upon the mathematical and conceptual frameworks established here. You are now well-prepared for advanced physics studies.



MEGACOURSE

Learn Anything. Achieve Everything.

Track your progress, take the quizzes, and earn your certificate for this course at
megacourse.app/courses/yale-fundamentals-of-physics